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Spatial Interpolation of Underway Nutrient Data

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EXECUTIVE SUMMARY

This report provides details of spatial smoothing and interpolation methods applied to underway nutrient data collected by the Marine and Fisheries Resources Institute as part of the Port Phillip Bay Environmental Study. The final products of these processes are finely resolved spatial interpolations of nutrients over the Bay together with estimated nutrient pool sizes and associated standard errors. The nutrients of interest are: Chlorophyll a (Chla); Ammonium (NH_4); Nitrite (NO_3); Nitrate (NO_2); Phosphate (PO4); and Silicate (SiO4).

Underway nutrient samples were collected approximately once a month for two years (May 1993 to March 1995). A total of 25 cruises were sailed with between 10,000 and 18,000 samples collected and analysed on each occasion. The total number of raw data values used in this report exceeds 343,000.

High sampling frequencies (typically 0.2Hz) resulted in data-dense transects that had low spatial coverage of the Bay. Although there were good reasons for this design, it nevertheless resulted in data that was not particularly conducive to Bay-wide interpolation. Another characteristic of the raw data was high frequency oscillations over short (10-20m) spatial scales. These were possibly both real (transient phenomenon in the water column) and artificial (artefacts of sampling and recording processes). In any event, since our interest centred on the characterisation of the local behaviour at spatial scales not less than 500m, there was a requirement to filter the raw data prior to interpolation. This was done using a robust kernel smoother in two dimensions.

The suitability of a number of interpolation methods was investigated during the course of this study including Kriging, inverse distance weighting, splines and triangulated irregular networks. The method we finally adopted was completely regularised splines with tension (CRS). This method is both flexible and robust and affords a high degree of control over the amount of local smoothing by adjustment of a tension parameter.

We found it necessary to aggregate the data on an annual basis so as to provide adequate spatial coverage of the Bay prior to interpolation. The interpolated nutrient concentrations obtained from this data were coupled with interpolated bathymetric data to arrive at final nutrient pool sizes over two consecutive twelve-month periods. Interpolation methods were also used to provide an approximate 95% upper confidence surface for the nutrient concentrations, which enabled us to attach a measure of uncertainty in the reported nutrient pool sizes.

An added challenge in this project was the requirement to obtain monthly estimates of nutrient pool sizes over the Bay. For reasons already noted, direct interpolation of the sparse transect data was not feasible. Instead, nutrient concentrations in empty grid cells which were more than 5km from a cruise path were estimated prior to interpolation. In this way, we were able to overcome the lack of spatial coverage for individual cruises. The actual and estimated data was then subjected to the CRS interpolation procedure to provide estimates and standard errors of nutrient pools on a monthly basis.

Finally, in order to dynamically represent changes in nutrient status in the Bay over time, we used sophisticated computer animation techniques to digitally combine sequences of colour-coded maps of nutrient concentrations.

A compressed video file has been provided on the Port Phillip Bay Environmental Study Technical Reports CDRom. Technical description and instruction for running the animation are provided in Appendix D.

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1. INTRODUCTION

Port Phillip Bay is the largest water body in Victoria, Australia. It is on the doorstep of Melbourne, the second largest city in Australia, and plays an important role in the social and economic activities of the state of Victoria. Additionally, commercial and recreational fishing and tourism are important activities, which rely on the sustained health of the Bay. The Bay receives waste discharges and nutrient inputs from many points, most significantly from the Yarra River to the north and the Western Treatment Plant (WTP) on the west coast.

The Port Phillip Bay Environmental Study, a 4 year $12M project, was established to assess the current status of the Bay, to investigate the biological processes occurring within the Bay and to provide recommendations on how to best utilize and manage the Bay in the future. Some 47 research tasks were conducted by over 30 research bodies from government agencies, universities and independent consultants in Australia and overseas.

The provision of sound estimates of the nutrient concentrations in Port Phillip Bay is an important component of Nutrient Task N2.1 of the Port Phillip Bay Environmental Study. It is particularly important to gain an understanding of the effects of both human behaviour, such as waste water inputs, and natural forces such as water movement in the Bay, on water quality, flora and fauna. The main issues to be addressed in this component of the task are (1) the interpolation of the distributions of nutrient concentrations over the entire Bay and (2) the estimation of the nutrient budgets over time. The nutrients of interest are Chlorophyll a (CHLa), Ammonium (NH_4), Nitrite (NO_2), Nitrate (NO_3), Phosphate (PO_4) and Silicate (SiO_4).

Sampling procedure

During a two year period, from May 1993 to March 1995, approximately 343,000 sub-surface water samples were collected along transects. On-broad analyses were conducted for six nutrients and these results were stored electronically. A total of 25 cruises were sailed over the two year period, with a 4 week interval between cruises. The number of samples collected in each cruise was between 10,000 and 18,000. We will often use month to refer to cruise to emphasize time interval. In 1993, almost all cruises followed the coast. Subsequent cruises included greater coverage of the central area of the Bay. Sampling along transects was approximately every 5 seconds which translated to approximately a 10 metre interval between consecutive samples. Other relevant variables recorded are longitude, latitude, temperature and salinity. For more details about the sampling procedure, see Longmore *et. al.* (1996).

Various approaches can be used to interpolate the spatial distributions of nutrients and to estimate nutrient pool sizes over the Bay. Longmore *et. al.* (1996) estimated nutrient pool sizes for individual cruises by multiplying the mean concentrations of the sub-surface samples by the water volume of the entire Bay. Although simple to implement, this is a rather crude approach. Furthermore, the method is susceptible to large biases because it does not capture the localised variability, which may be substantial. Interpolation techniques may help redress this deficiency by accommodating natural gradients in nutrient concentrations. Longmore *et. al.* (1996) also applied Inverse Distance Weighting (IDW) to the raw sub-surface data (5 second interval) for individual cruises and for data averaged over one minute. A number of concerns have been raised in the literature about the suitability of IDW as a spatial interpolator and these are discussed in Section 3.1. A fundamental problem for the current interpolation exercise is the exceedingly high density of information *along* a transect (approximately every 10 metres), but low spatial coverage *over* the entire Bay for individual cruises. Figure 1 shows the path of a typical cruise.

The problem of over-intensive sampling becomes more critical when interpolating *between* transects since the inter-sample distance is too small in relation to the size of grid cells used for interpolation. In fact, we use the interpolated concentration at the centre of a 500 by 500 metre cell as representative of this cell for the estimation of the Bay-wide nutrient pool sizes. We note that a 500 by 500 metre cell may contain up to 70 observations. By using the original observations, the interpolated concentration at the centre may not adequately represent the concentration of that cell due to the large variation in the concentration at small distances. Thus, there is a requirement to smooth the data prior to interpolation. The first part of Section 2 gives the details of the smoothing technique adopted in this report.

To overcome the low spatial resolution between transects of individual cruises, monthly cruise data was pooled. Initially, this was done over seasons, although considerable gaps remained (see Figure 2). It was therefore necessary to aggregate cruise data on an annual basis. Figure 3 shows a plot of sample points for the cruises sailed between May 1993 and April 1994. The annual data set provides sufficient spatial coverage over the entire Bay. When interpolating annual distributions of nutrients, Longmore *et. al*. (1996) pooled the monthly data together and then applied a double-spline method (Longmore, Per. Com.) of interpolation to the raw data. We will show that this procedure results in biased estimation of annual mean concentrations and we have therefore adopted an alternative procedure which provides unbiased estimation of annual mean nutrient concentrations. Details of this procedure are given in Section 2.2.

In section 3, we review two frequently used interpolation methods, Kriging and inverse distance weighting, although for reasons given later in this report, we adopted the method of completely regularised splines (CRS) with tension, as described by Mitásová and Mitás (1993). Some statistical considerations associated with the CRS procedure are given in Section 5.

Two disjoint years, May 1993 to April 1994 and May 1994 to March 1995, were used as the basis for examining the change in nutrient pools over time. The sounding samples provided by Royal Australian Navy and Hydrographic Office (Figure 4) were interpolated at the same spatial scale as used for nutrient interpolations. The combination of interpolated nutrient distributions and bathymetric distribution enabled us to calculate nutrient pools over the entire Bay. Nutrient pool sizes were calculated for the Bay as a whole and for each of the seven environmental zones used in the Phase One study (MMBW/FWD, 1973). These zones are illustrated in Figure 5. The upper bounds of the approximate 95% confidence intervals for nutrient pool sizes can also be estimated by combining the interpolated distributions of standard errors and bathymetric distribution.

As already noted, the design of a cruise transects resulted in insufficient spatial coverage over the Bay for successful interpolation on a monthly basis. Instead of direct interpolation, we estimated nutrient concentrations in those empty grid cells which exceeded a predetermined distance from a cruise path prior to interpolation. The technical details for interpolating monthly nutrient distributions are given in Section 4. Monthly nutrient pool sizes can be estimated in the similar manner as annual nutrient pool sizes. Section 6 gives all the statistical results.

Finally, sequences of color-coded images of nutrient concentrations over the entire Bay were digitally combined to produce animated video images.

A compressed video file has been provided on the Port Phillip Bay Environmental Study Technical Reports CDRom. Technical description and instruction for running the animation are provided in Appendix D.

2. DATA PROCESSING

2.1. Spatial Smoothing

Recall that the distance between two adjacent sample locations along a cruise transect is approximately 10 metres. A basic property of the completely regularised splines (CRS) interpolation method used in this report is that the fitted surface will honour the actual data (i.e. pass through all sample points). Given that we are attempting to characterise a smooth trend surface, it will be necessary to filter out high frequency oscillations in the transect data prior to fitting the surface. To this end, we have applied a kernel-smoother as described below.

Assume that z_i is the value of a measurement at point $\mathbf{x}_i = (x_i^{[1]}, x_i^{[2]}, \cdots, x_i^{[d]})^T$ in a d-dimensional space, $(i = 1, 2, \cdots, n)$. A kernel estimate of the measurement, $z(\mathbf{x})$, at point $\mathbf{x} = (x^{[1]}, x^{[2]}, \cdots, x^{[d]})^T$ is

$$\hat{z}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} w(\mathbf{x}, \mathbf{x}_i) z_i,$$

where

$$w(\mathbf{x}, \mathbf{x}_i) = \frac{1}{h} K\left[\frac{\mathbf{x} - \mathbf{x}_i}{h}\right] \Big/ \sum_{j=1}^{n} \frac{1}{nh} K\left[\frac{\mathbf{x} - \mathbf{x}_j}{h}\right] .$$

The kernel function K is assumed to be symmetric about zero and integrates to 1. h is called the window size or bandwidth of the smoothing procedure and controls the extent of local smoothing. For a more complete discussion on kernel smoothing see Wand and Jones (1995).

The smoothed value $\hat{z}(\mathbf{x})$ is affected by the kernel function K and the window size h. After examining the performances of different kernel functions, Wand and Jones (p.31 and p.104, 1995) pointed out that the choice of the shape of the kernel function is not particularly important. Based primarily on considerations of simplicity we used the kernel function

$$K[\mathrm{x}] = \begin{cases} 1, & \text{if } |x^{[l]}| \le \tfrac{1}{2} \quad \forall\, l, \\ 0, & \text{Otherwise,} \end{cases}$$

which is the uniform distribution with compact support on $[-\tfrac{1}{2}, \tfrac{1}{2}]$ in each dimension. In this case, the smoothed value $\hat{z}(\mathbf{x})$ is simply the average of the observations within the window. That is

$$\hat{z}(\mathbf{x}) = \frac{1}{N[\#(\mathbf{x})]} \sum_{i;\, i \in \#(\mathbf{x})} z_i,$$

where $\#(\mathbf{x}) = \{i; |x_i^{[l]} - x^{[l]}| \le h, \forall\ l\}$ indexes all the sample points which fall in the window of size *h*. $N[\#(\mathbf{x})]$ is the number of elements in $\#(\mathbf{x})$. We call the smoother with the above kernel function as the *mean smoother*.

The choice of the window size is very important in kernel smoothing. Using a small window size in the averaging process results in a coarse estimate of $z(\mathbf{x})$ because the smoothing process at each point is based on relatively few observations. Such an estimate is said to be *undersmoothed*. On the other hand, a too large window size gives an *oversmoothed* estimate of $z(\mathbf{x})$. Wand and Jones (Section 4.7, 1995) discuss the window size selection in more detail. In our application, the mean smoother is used to obtain a representative nutrient concentration at each grid cell. Therefore, the window size should be the same as that for the grid cell. However, to preserve smoothness, a larger window size was used.

For plotting purposes it is necessary to project the nutrient distributions onto a rectangular region which is then partitioned into small regular cells. These small grid cells are identified as $G_1, G_2, \cdots, G_i, \cdots$. Figure 6 illustrates how the partitioning was performed. Each of these cells was assigned an indicator variable to indicate whether it was inside or outside the Bay. All cells inside the Bay are called interior cells. Boundary cells were then defined as the collection of cells outside the Bay and have at least one boundary (or point) in common with an interior cells. In Figure 6, the blue-shaded regions illustrate the interior cells, while the yellow-shaded regions illustrate the boundary cells. We only interpolate values for interior cells. Computational details of the smoothing process used for the nutrient data are described below.

Assume that the number of cruises is *M* (rather than 25) for generalization. Let $(x_{mj}, y_{mj}, z_{mj}^{(k)})$ denote the Easting, Northing and value of the k^{th} nutrient from the raw data of cruise *m*. Let (X_i, Y_i) denote the Easting and Northing of the center of regular grid cell G_i. Assume that the window size is *h*. The smoothed value of $Z_{mi}^{(k)}$, the k^{th} nutrient for cruise *m* in cell G_i is defined as

$$\hat{Z}_{mi}^{(k)} = \frac{1}{N[\#_{mi}^{(k)}]} \sum_{j;\, j \in \#_{mi}^{(k)}} z_{mj}^{(k)},$$

where $\#_{mi}^{(k)} = \{j; |x_{mj} - X_i| \le h, |y_{mj} - Y_i| \le h, \text{ and } z_{mj}^{(k)} \ge 0\}$, which indexes all the sample points that fall within G_i and neighboring cells, and whose observations for the k^{th} nutrient are not missing. $N[\#_{mi}^{(k)}]$ is the number of the elements in $\#_{mi}^{(k)}$. We let $Z_{mi}^{(k)}$ be missing if $\#_{mi}^{(k)}$ is empty.

For each cell, we also define the estimated standard deviation (E.S.D.) and the estimated standard error (e.s.e.) of $Z_{mi}^{(k)}$ as the following,

$$E.S.D.(\hat{Z}_{mi}^{(k)}) = \left[\frac{1}{N[\#_{mi}^{(k)}]-1} \sum_{j;\, j \in \#_{mi}^{(k)}} (z_{mj}^{(k)} - \hat{Z}_{mi}^{(k)})^2 \right]^{1/2}$$

and

$$e.s.e.(\hat{Z}_{mi}^{(k)}) = E.S.D.(\hat{Z}_{mi}^{(k)}) \Big/ \sqrt{N[\#_{mi}^{(k)}]},$$

for $N[\#_{mi}^{(k)}] > 1$.

2.2. Data Pooling

To obtain smoothed values from pooled monthly sub-surface data, one approach, as suggested by Longmore *et. al.* (1996), is to simply pool the raw data and apply the above smoothing technique to the pooled data set. However, as shown in Appendix A, biased estimation of the true mean concentration of a cell occurs when the smoothing is performed in this manner. An alternative approach is to pool monthly smoothed data and then take the average for each regular cell. That is,

$$\hat{Z}_i^{(k)} = \frac{1}{M} \sum_{m=1}^{M} \hat{Z}_{mi}^{(k)}$$

for all non-missing $Z_{mi}^{(k)}$. The standard error can be estimated by

$$e.s.e.(\hat{Z}_i^{(k)}) = \frac{1}{M} \sqrt{\sum_{m=1}^{M} \left[e.s.e.(\hat{Z}_{mi}^{(k)}) \right]^2} \ .$$

In fact, $\hat{Z}_{mi}^{(k)}$ and $e.s.e.(\hat{Z}_i^{(k)})$ are the estimates of the mean and its standard error over the $m = 1, \cdots, M,$ cruises. However, for almost all interior cells G_i, there are many cruises for which some of the $\{ Z_{1i}^{(k)}, Z_{2i}^{(k)}, \cdots, Z_{Mi}^{(k)} \}$ are missing. In this case, modifications to the estimation procedure for $\hat{Z}_{mi}^{(k)}$ and $s.e.(\hat{Z}_i^{(k)})$ are

$$\hat{\hat{Z}}_i^{(k)} = \frac{1}{\overline{M}_i} \sum_{m=1}^{\overline{M}_i} \hat{Z}_{(m)i}^{(k)} \ ,$$

and

$$e.s.e.(\hat{\hat{Z}}_i^{(k)}) = \frac{1}{\overline{M}_i} \sqrt{\sum_{m=1}^{\overline{M}_i} \left[e.s.e.(\hat{Z}_{(m)i}^{(k)}) \right]^2} \ ,$$

respectively, where $\overline{M}_i$ is the number of cruises in which the smoothed value $\hat{Z}_{mi}^{(k)}$ is not missing in G_i. $\{\hat{Z}_{(1)i}^{(k)}, \hat{Z}_{(2)i}^{(k)}, \cdots, \hat{Z}_{(\overline{M}_i)i}^{(k)}\}$ is the collection of $\{\hat{Z}_{1i}^{(k)}, \hat{Z}_{2i}^{(k)}, \cdots, \hat{Z}_{Mi}^{(k)}\}$, derived from mean smoothing, containing no missing values. Appendix A establishes that, if the smoothed values $\{\hat{Z}_{mi}^{(k)}, m = 1, \cdots, M\}$ are randomly missing, then the above modifications still give unbiased estimates of the mean and its standard error over the M cruises.

3. INTERPOLATION METHODS

3.1. Review of Common Approaches

Two candidate interpolation methods are Kriging and inverse distance weighting (IDW). Both of these methods are discussed by Journel and Huijbregts (1978) and Cressie (1991). The basis of Kriging is to estimate the covariance of measurements between locations as a function of their separation and orientation. This characterisation of spatial dependency is then used to determine an 'optimal' set of weights applied to the sample data in the neighbourhood of an unsampled location. It has been shown that, under certain assumptions, Kriging is a Best Linear Unbiased Estimator (BLUE).

A critical assumption of Kriging is second-order stationarity, i.e.

$$\text{Centered covariance,} \quad E\{Z(\mathbf{x}+\mathbf{h})Z(\mathbf{x})\} - \{E[Z(\mathbf{x})]\}^2 = C(\mathbf{h});$$
$$\text{Variogram,} \quad E\left\{\left[\, Z(\mathbf{x}+\mathbf{h}) - Z(\mathbf{x})\right]^2\right\} = 2\gamma(\mathbf{h}),$$

where $Z(\mathbf{x})$ is the value of the measurement at the point **x**. Either of these second-order moments are assumed known and to depend only on **h**, the lag or separation (second-order stationarity). Kriging was deemed to be unsuitable for the present study as both current and previous data suggest that the processes involved violate this second-order stationary requirement.

Inverse distance weighting is discussed by Isaaks and Srivastava (1989), and is simpler to use and understand than Kriging. IDW estimates the value at a given point, using a weighted average, i.e.

$$Z(\mathbf{x}) = \sum_{j=1}^{N} w(\mathbf{x}, \mathbf{x_j}) z_j ,$$

where $w(\mathbf{x}, \mathbf{x}_j)$ is a non-increasing function of the distance $d(\mathbf{x}, \mathbf{x}_j)$ between the target point **x** and sample point $\mathbf{x}_j$ and satisfies the constraint $\sum_{j=1}^{N} w(\mathbf{x}, \mathbf{x}_j) = 1$. We call $w(\mathbf{x}, \mathbf{x}_j)$ the weight function of z_j. Inverse distance weighting is a type of kernel smoother, but applied to the whole area spanned by the data.

The simplest weight function is inversely proportional to distance:

$$w(\mathbf{x}, \mathbf{x}_j) = \frac{1/d(\mathbf{x}, \mathbf{x}_j)}{\sum_{k=1}^{N} 1/d(\mathbf{x}, \mathbf{x}_k)}.$$

A further generalization adopts weights as inversely proportional to a power of the distance:

$$w(\mathbf{x},\mathbf{x}_j)=\frac{\left[1/d(\mathbf{x},\mathbf{x}_j)\right]^p}{\sum_{k=1}^{N}\left[1/d(\mathbf{x},\mathbf{x}_k)\right]^p}.$$

The Surfer® (Golden Software, Inc.) names the case p=2 and p=3 as inverse distance squared and inverse distance cubed, respectively.

This is not an efficient method if the number of samples is large. As an alternative, packages such as ARC/INFO® (Environmental Systems Research Institute, Inc.) and CRS/PC® (Clinical Research System, Inc.) only include observations within a *search radius* around the point of interest. CRS/PC® also uses the weighting function

$$w(\mathbf{x},\mathbf{x}_j)=\frac{\left[\dfrac{d_0-d(\mathbf{x},\mathbf{x}_j)}{d(\mathbf{x},\mathbf{x}_j)}\right]^2}{\sum_{k=1}^{n}\left[\dfrac{d_0-d(\mathbf{x},\mathbf{x}_k)}{d(\mathbf{x},\mathbf{x}_k)}\right]^2},$$

where n is the number of samples inside a radius d_0 about the point **x**, designed to assign more weight to nearby observations.

Weber and Englund (1994) have compared ordinary kriging and inverse distance weighting, pointing out that inverse distance estimators are very sensitive to some types of samples, to the search radius used in the estimate and to the choice of weights. Notice that the choices of weights and search area are quite arbitrary and great care should be exercised in applying this method. Notice further that inverse distance weighting can only interpolate the values within the area spanned by the observed data rather than the whole Bay. For these reasons we have chosen not to pursue IDW any further.

3.2 Completely Regularized Spline with Tension

We have adopted a spline function method proposed by Mitásová and Mitás (1993), which follows the fundamental work of Talmi and Gilat (1977). A brief description of this approach follows.

Talmi and Gilat (1977) suggested the smoothness of an interpolation function $z=g(\mathbf{x})$, $\mathbf{x}=(x^{[1]},x^{[2]},\cdots,x^{[d]})^T$, in a region Ω of d-dimensional space, can be measured by the following smooth seminorm $E^2(g)$,

$$E^2(g) = \sum_{\alpha} B_\alpha \iint_\Omega \left[\frac{\partial^{|\alpha|}}{\partial^{\alpha_1} x^{[1]} \cdots \partial^{\alpha_2} x^{[d]}} g(\mathbf{x}) \right]^2 dx^{[1]} \cdots dx^{[d]},$$

where $\alpha_1, \cdots, \alpha_d$ are nonnegative integers with $|\alpha| = \sum_i \alpha_i$ and the $\{B_\alpha\}$ are nonnegative constants. Given a set of samples $\{(\mathbf{x}_i, z_i) \mid i = 1, 2, \cdots, N\}$, the function $z = g(\mathbf{x})$ should satisfy the following conditions:

(1) $g(\mathbf{x}_i) = z_i$, for all sample points $\{(\mathbf{x}_i, z_i) \mid i = 1, 2, \cdots, N\}$,

(2) $E^2(g)$ is a global minimum.

Mitásová and Mitás (1993) specified and used

$$B_\alpha = \begin{cases} 0, & |\alpha| = 0 \\ \dfrac{C(\alpha)}{\varphi^{2|\alpha|}(|\alpha|-1)!}, & |\alpha| > 0 \end{cases},$$

where

$$C(\alpha) = \frac{|\alpha|!}{\alpha_1! \cdots \alpha_d!}$$

and φ is a relative reciprocal weight, called the tension parameter. B_α is the so-called completely regularized spline with tension parameter φ. The resulting interpolation function is

$$z = g(\mathbf{x}) = a + \sum_{i=1}^{N} \lambda_i \; R[\mathbf{x}, \mathbf{x}_i]$$

where

$$R[\mathbf{x}, \mathbf{x}_i] = -\left\{ \ln\left[\left(\frac{\varphi\, r_i}{2} \right)^2 \right] + E_1\left[\left(\frac{\varphi\, r_i}{2} \right)^2 \right] + C_E \right\}$$

with exponential integral function $E_1(\cdot)$ (see Gautschi and Cahill, 1970) and the Euler constant $C_E = 0.577215\cdots$. Here $r_i = [\sum_{j=1}^{d} (x^{[j]} - x_i^{[j]})^2]^{1/2}$. The coefficients a and $\{\lambda_i\}$ can be obtained by solving the following system of linear equations

$$g(\mathbf{x}_i) = z_i, \qquad i = 1, \cdots, N,$$
$$\sum_i \lambda_i = 0.$$

Note firstly that the interpolation at a sample point is the same as its value, i.e. the surface passes through all sample points. Secondly, the surface is smooth in the high order derivatives with respect to **x**.

4. INTERPOLATIONS FOR INDIVIDUAL CRUISES

The design of cruise transects deliberately focused (at least initially) on the shallower regions of the Bay, since this is where the nutrient concentrations are most variable. However, as already noted, this sailing pattern resulted in insufficient spatial coverage of the Bay for successful interpolation on a monthly (cruise by cruise) basis. Nevertheless, monthly nutrient pool sizes were required for the calibration of the integrated ecological model of the Bay (Task G8/9). To overcome insufficient spatial coverage over the Bay, the following procedure was adopted.

The mean smoothing and pooling methods described in Section 2 were first applied to the sub-surface samples of the 25 cruises combined. This produced a smoothed average over all cruises that had sufficient spatial coverage over the Bay. The CRS procedure was then applied to this smoothed data to obtain interpolated average surface nutrient distributions of all cruises. Let $\tilde{Z}^{(k)}_{\mathrm{all},i}$ be the interpolated average concentration of the k^{th} nutrient at cell G_i over all cruises.

For Cruise m, we define a *remote* set as the collection of cells that exceed a predetermined distance from the nearest smoothed sample locations. That is

$$U^{(k)}_{m,r} = \{ G_i \,;\ \min_{j,\, j \in \#^{(k)}_m} [(|X_i - X_j|^2 + |Y_i - Y_j|^2)^{1/2}] > r \},$$

where

$$\#^{(k)}_m = \{ j\,;\, \hat{Z}^{(k)}_{mj} \quad \text{is not missing } \}$$

indexes the smoothed sample locations, and r is a positive constant. For each cell $G_i \in U^{(k)}_{m,r}$, we intended to use the interpolated average concentration of all cruises, $\tilde{Z}^{(k)}_{\mathrm{all},i}$, as the estimated nutrient concentration at this remote cell. For each cruise, since nutrient concentrations vary between environmental zones we cannot simply use the interpolated average concentration of all cruises, $\tilde{Z}^{(k)}_{all,i}$, as the estimated nutrient concentration for each remote cell $G_i \in U^{(k)}_{m,r}$. Instead we must adjust for this source of variation. We can do this quite simply by adding a correction factor, $c^{(k)}_m$ say, which is the average difference between the smoothed nutrient concentration for cruise m and the average over all cruises. Formally, for zone *A* the correction factor is

$$c^{(k)}_{m,A} = \frac{1}{N[\#^{(k)}_{m,A}]} \sum_{j,\, j \in \#^{(k)}_{m,A}} (\hat{Z}^{(k)}_{mj} - \tilde{Z}^{(k)}_{\mathrm{all},j}),$$

where

$$\#^{(k)}_{m,A} = \{ j, | G_j \in U^{(k)}_{m,r} \cap A \}.$$

The missing values in each remote cell in zone *A* were then replaced by

$$\hat{Z}_{mi}^{(k)} = \widetilde{Z}_{\text{all},i}^{(k)} + c_{m,A}^{(k)} \quad \text{if} \quad G_i \in U_{m,r}^{(k)} \cap A,$$

Repeating this procedure for each environmental zone enabled sufficient spatial coverage to be achieved. The CRS method was then used to interpolate the monthly nutrient distributions.

5. STATISTICAL CONSIDERATIONS

5.1. Choice of tension parameter and minimum cell number used

In our application, smoothness is an important requirement of the interpolated nutrient distributions. This is because abnormally high or low values of nutrient concentrations in the Bay will tend to be smoothed out by natural forces such as the prevailing currents and winds, and diffusion within the water column. The CRS approach is well-suited to the interpolation of nutrient data as it offers a great degree of control over the smoothness of the interpolated surface. The determination of an appropriate value of the smoothing parameter (φ) is necessarily subjective, although reasonable values are readily determined by trial and error.

Given the sample observations, the rigidity of the interpolation function increases as the tension parameter increases. There is a threshold φ_0 such that, for $\varphi \le \varphi_0$, the interpolated surface will tend to fluctuate much more rapidly and have some overshooting near the extreme points. The effect of tension on the interpolation is illustrated in Figure 7. The value of the tension parameter was determined by the stability of the interpolation functions and the goodness of fit, which were assessed via cross-validation. This was done by randomly deleting a part of the smoothed values and then interpolating the deleted part by using the CRS procedure based on the remaining data. In our application, we found that a reasonable good fit can be achieved by choosing φ >20. However, the overshooting problem is not ameliorated for φ <40. The interpolation results are unchanged for φ >120. Thus we restrict ourselves to values of tension in the range of 40 to 120.

As pointed out by Mitásová and Mitás (1993), this method is prohibitively slow and possibly unstable for large data sets like ours. They recommended an algorithm for segmented processing given by Franke (1982). The algorithm only utilises the sample data in the $B \times B$ cells surrounding the cell to be interpolated. B was chosen such that the number of samples used was greater than a specified minimum, $\beta_{\min}$.

Note that both φ and $\beta_{\min}$ are determined by sampling patterns rather than sample values. After trying different combinations of φ and $\beta_{\min}$, we finally chose $\varphi = 70$ and $\beta_{\min}$=100. We used these values for all nutrients and time periods. For the interpolation of sounding samples, values $\varphi = 45$ and $\beta_{\min}$=80 were chosen.

For mean smoothing, a smoothed value cannot be considered stable if $N[\#_{mi}^{(k)}]$ is too small, so any smoothed value based on less than 10 samples was set to missing.

5.2. Boundary Value Considerations.

Notice that the water depth at each location converges to zero when the location approaches the boundary of the Bay and is identically equal to zero on the boundary. The original sounding data was augmented prior to interpolation to reflect the fact that water depth is zero on the Bay's boundary. Note that the values across the opening to Bass Strait were set to missing. The zero value on the boundary behaves as a control for the interpolation process. Thus, the depth tends towards zero as the cells approach the boundary. The only problem with setting boundary values to zero is that we did not set the boundary across the entrances to the Yarra River to missing due to lack of information. However, this does not affect our results to any large extent, particularly the total nutrient pools in the Bay.

5.3. Interval Estimates.

The CRS procedure can also be used to interpolate the distributions of the upper bound of the 95% confidence interval of nutrient concentrations. We investigated two alternatives:

(1) Interpolate the standard error $e.s.e.(\hat{Z}_i)$ and then define the upper limit of the 95% confidence interval to be $\tilde{Z} + 1.96 * e.s.e.(\tilde{Z})$;

(2) Interpolate the upper bound of the 95% confidence interval directly by using the samples at $\{(X_i, Y_i)\}$ with the values $\{\hat{Z}_i + 1.96 * e.s.e.(\hat{Z}_i)\}$.

It is shown in Appendix B that the resulting interpolations are identical.

6. RESULTS

All the computations were performed in GAUSS® (Aptech Systems, Inc.). Appendix C lists all details of the GAUSS® routines used in this report. All images were produced in SAS® (SAS Institute Inc.).

Approximately 2,300 hours of computer time on a 120MHz Pentium PC were used to accomplish all the interpolations, while image processing and editing took about 160 hours. About 80 hours of effort were required to create the video files.

We chose the rectangle with Easting ranging from 265000 to 337500 and Northing ranging from 5749000 to 5815500, thus covering the whole Bay. This rectangle was then partitioned into 500m×500m small regular cells, giving 145 (Easting) by 133 (Northing) or 19,285 cells in total. These small cells are identified as $G_1, G_2, \cdots, G_i, \cdots, G_{19285}$. It was found that 7,696 cells are inside the Bay. In the mean smoothing procedure, a window size $h = 750\text{m}$ was chosen, since this was a convenient multiple (3×3) of the basic cell size. See Figure 6 for an illustration.

The 7,696 interior cells of size 500×500m had a surface area of 1,924 km^2. The interpolated bathymetry distribution is shown in Figure 8. The estimated water volume is 24.478 km^3 with maximum depth of 25 metres. The central area, Zone 6, contains 70% of the total water volume, while the opening area to the sea, Zone 1, contains only 9% of the total water volume.

The surface plots of the spatial distribution of yearly average for each of the nutrients are given in Figure 9 (May 1993 to April 1994) and Figure 10 (May 1994 to March 1995). We call these two disjoint years the first and second year, respectively. Year one uses the first 13 cruises while year two uses the remaining 12 cruises. Comparisons can be made between these two consecutive years. It was found that Silicates are more evenly distributed over the Bay than other nutrients, except around the entrance of the Yarra River (Zone 7). The Phosphate concentrations are higher in the north than the south, while Chlorophyll *a*, Ammonium, Nitrite and Nitrate have higher concentrations along the Northern Coast (Zones 3 and 4). For all Nutrients, the pool sizes are smaller in the second year. Table 1 gives the yearly averages of nutrient pool sizes and the 95% upper bounds for the whole Bay, and for each of the environmental zones. Figures 11 and 12 show the distributions of standard errors of the estimated nutrient concentrations. It can be seen that the standard errors for Ammonium and Nitrate are large in the north-west of the Bay. All the other nutrients have quite stable standard errors, except for the Silicates at the entrance of the Yarra River. The 95% upper bounds are also given in Table 1.

The interpolated nutrient distributions of all cruises and their standard errors are given in Figures 13 and 14. The pool sizes and their 95% upper bounds are given in Table 2. To interpolate the monthly nutrient distributions and their standard errors, we used the predetermined distance $r = 5\text{km}$. Figures 15 to 20 show the interpolated monthly surface distributions for each nutrient. The associated distributions of standard errors are given in Figures 21 to 26. Tables 3 to 8 give the estimated monthly pool sizes and the corresponding 95% upper limits for each nutrient in each environmental zone, and in the whole Bay.

7. COMPUTER ANIMATIONS

To dynamically evaluate the change in nutrient status over time, computer animation files were constructed. The original BMP formatted images were read into Mathcad® (MathSoft Inc.) and then converted into AVI formatted video files. Using a technique known as morphing, another animation file was created, showing all six nutrient distributions in a single frame.

A compressed video file has been provided on the Port Phillip Bay Environmental Study Technical Reports CDRom. Technical description and instruction for running the animation are provided in Appendix D.

8. ACKNOWLEDGMENTS

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9. APPENDICES

APPENDIX A

COMPARISON OF TWO SMOOTHING PROCEDURES APPLIED TO AGGREGATED DATA SET.

In this appendix, we establish that

(1) applying the mean smoothing technique to the simple combination of monthly raw data gives a biased estimate of the annual average;

(2) the average of the monthly smoothed values is an unbiased estimate of the annual average if the missing values occur randomly among months.

Suppose there are M random variables $\{ X_m , m = 1, 2, \cdots, M \}$ with mean μ_m and variance σ_m^2, and there are n_m observations $\{x_{mi}, i = 1, \cdots, n_m\}$ for variable X_m. The parameter of interest is the unweighted average of $\mu_1, \cdots, \mu_M$, i.e.

$$\mu = \frac{1}{M} \sum_{i=1}^{M} \mu_m .$$

The simple average of the raw data is

$$X^{(1)} = \sum_{m=1}^{M} \sum_{i=1}^{n_m} x_{mi} \Big/ \sum_{m=1}^{M} n_m ,$$

which is the mean smoother applied to the simple collection of the raw data for all the variables. The expected value of $X^{(1)}$ is

$$E[X^{(1)}] = \sum_{m=1}^{M} n_m \mu_m \Big/ \sum_{m=1}^{M} n_m ,$$

which is a weighted average of $\mu_1, \cdots, \mu_M$.

It can be seen that $E[X^{(1)}] \neq \mu$ unless the numbers of observations n_i are all equal (i.e. $n_i = n_j \ \forall i, j$), or the mean values μ_m are all equal (i.e. $\mu_i = \mu_j \ \forall i, j$). Therefore, $X^{(1)}$ is generally a biased estimate of μ.

Another estimator for μ, similar to that used in this report, is determined by averaging the sample mean of each variable. That is,

$$X^{(2)} = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{n_m} \sum_{i=1}^{n_m} x_{mi} = \frac{1}{M} \sum_{m=1}^{M} \overline{X}_m .$$

It is easy to show that $X^{(2)}$ is an unbiased estimate of μ.

In practice, observations for some variables will be missing. In this case, we modify the estimate $X^{(2)}$ to

$$X^{(3)} = \frac{1}{\overline{M}} \sum_{m=1}^{\overline{M}} \overline{X}_{(m)} ,$$

where $\overline{M}$ is the number of variables which have at least one observation and (m) indexes these variables.

If we further assume that the missing values occur randomly among all M variables, then the modified estimate $X^{(3)}$ is still unbiased estimate of μ. We prove this as follows.

Suppose that $\{X_{(1)}, X_{(2)}, \cdots, X_{(\overline{M})}\}$ is any combination from $\{X_1, X_2, \cdots, X_M\}$. There are $C_M^{\overline{M}}$ different combinations, each of which is equally likely. Thus,

$$E[X^{(3)}] = \frac{1}{k} \cdot \frac{1}{C_M^{\overline{M}}} \sum (\mu_{(1)} + \cdots + \mu_{(k)}) ,$$

where the summation is taken over all the $C_M^{\overline{M}}$ different combinations. Because all the μ_m appear the same number of times in the combinations, and each of them appears $C_M^{\overline{M}} \cdot \overline{M}/M$ times, we then have

$$E[X^{(3)}] = \frac{1}{\overline{M}} \cdot \frac{C_M^{\overline{M}} \cdot \overline{M} / M}{C_M^{\overline{M}}} (\mu_1 + \mu_2 + \cdots + \mu_M) = \mu .$$

Thus $X^{(3)}$ is unbiased estimate of μ.

APPENDIX B

A UNIQUENESS PROPERTY OF COMPLETELY REGULARISED

SPLINES WITH TENSION

In this appendix, we establish a uniqueness of spline interpolation, given by Talmi and Gilat (1977), which is applicable to the completely regularized spline with tension proposed by Mitásová and Mitás (1993).

For a given function $g(\mathbf{x})$, $\mathbf{x}=(x^{[1]},x^{[2]},\cdots,x^{[d]})$, in a region Ω of d-dimensional space, the smoothness of $g(\cdot)$ is measured via the following smooth seminorm $E(g)$ defined by

$$E^2(g)=\sum_{\alpha} B_\alpha \int\cdots\int_\Omega \left[\frac{\partial^{|\alpha|}}{\partial^{\alpha_1}x^{[1]}\partial^{\alpha_2}x^{[2]}\cdots\partial^{\alpha_d}x^{[d]}}g(\mathbf{x})\right]^2 dx^{[1]}dx^{[2]}\cdots dx^{[d]} ,$$

where $\alpha=(\alpha_1,\alpha_2,\cdots,\alpha_d)$ is a multi-index with nonnegative integer components, with $|\alpha|=\sum_i \alpha_i$, and $\{B_\alpha\}$ are nonnegative constants.

Suppose that two studied phenomena are measured at discrete points $\mathbf{x}_i=(x_i^{[1]},x_i^{[2]},\cdots,x_i^{[d]})$, $i=1,2,\cdots,N$ in a certain region of a d-dimensional space, with values $z_i^{(1)}$ and $z_i^{(2)}$, respectively. The interpolation problem is formulated as follows:

Construct functions

$$z^{(l)}(\mathbf{x})=\sum_{k=1}^{\infty} A_{lk}h_k(\mathbf{x}),\ l=1 \text{ or } 2,$$

where $\{h_k\}$ is a series of basis functions, so that

(i) $z^{(l)}(\mathbf{x}_i)=z_i^{(l)}$ for all the given points of $\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_N$, and

(ii) $E^2\left[z^{(l)}(\mathbf{x})\right]=$ minimum.

We now show that:

for any two constants c_1 *and* c_2, *The function* $c_1z^{(1)}(\mathbf{x})+c_2z^{(2)}(\mathbf{x})$ *is the interpolation for the studied phenomenon* $c_1z_i^{(1)}+c_2z_i^{(2)}$ *at a given point* $\mathbf{x}_i=(x_i^{[1]},x_i^{[2]},\cdots,x_i^{[d]})$, $i=1,2,\cdots,N$.

Talmi and Gilat (1977) showed that $z^{(l)}(\mathbf{x})$ has the unique expression of the form

$$z^{(l)}(\mathbf{x}) = T_l(\mathbf{x}) + \sum_{i=1}^{N} \lambda_{l,j} R(\mathbf{x}, \mathbf{x}_i).$$

The 'trend' function $T_l(\mathbf{x})$ is given by

$$T_l(\mathbf{x}) = \sum_{j=1}^{M} a_{l,j} f_j(\mathbf{x})$$

where $\{f_j(\mathbf{x})\}$ is a set of linearly independent functions which have zero smooth seminorm. $R(\mathbf{x}, \mathbf{x}_i)$ is a radial basis function with an explicit form depending on the choice of $\{B_\alpha\}$. Once the basis functions $\{h_k\}$ are given, the coefficients $\{a_{l,j}\}$ and $\{\lambda_{l,j}\}$ are obtained by solving the following system of linear equations

$$\begin{aligned} z^{(l)}(\mathbf{x}_i) &= z_i^{(l)}, \qquad i = 1, 2, \cdots, N \\ \sum_{i=1}^{N} \lambda_{l,i} f_j(\mathbf{x}_i) &= 0, \quad j = 1, 2, \cdots, M. \end{aligned}$$

Let $\lambda_i = c_1\lambda_{1,i} + c_2\lambda_{2,i}$ and $a_l = c_1 a_{l,1} + c_2 a_{l,2}$. We see, by substitution, that

$$Z(\mathbf{x}) = T(\mathbf{x}) + \sum_{i=1}^{N} \lambda_i R(\mathbf{x}, \mathbf{x}_i),$$

with $T(\mathbf{x}) = \sum_{l=1}^{M} a_l f_l(\mathbf{x})$, and the conditions

$$\begin{aligned} z(\mathbf{x}_i) &= z_i, \qquad i = 1, 2, \cdots, N \\ \sum_{i=1}^{N} \lambda_i f_l(\mathbf{x}_i) &= 0, \quad l = 1, 2, \cdots, M \end{aligned}$$

are satisfied.

Thus, $z(\mathbf{x})$ is the interpolation function for the studied phenomenon $c_1 z^{(1)}(\mathbf{x}) + c_2 z^{(2)}(\mathbf{x})$ at the given point $\mathbf{x}_i = (x_i^{[1]}, x_i^{[2]}, \cdots, x_i^{[d]})$, $i = 1, 2, \cdots, N$ and $z(\mathbf{x}) = c_1 z^{(1)}(\mathbf{x}) + c_2 z^{(2)}(\mathbf{x})$. The uniqueness ensures that $z(\mathbf{x}) = c_1 z^{(1)}(\mathbf{x}) + c_2 z^{(2)}(\mathbf{x})$ is the only solution.

APPENDIX C

GAUSS® ROUTINES DEVELOPED FOR THIS REPORT

This appendix contains all the GAUSS® routines used in this report. Two routines, named `video` and `frame`, are not presented here. `video` controls the colour attribution displayed on the monitor and `frame` draws a frame at a predetermined position on the monitor. They can be obtained directly from the authors.

(C1) Mean Smoothing Routine

The following routine performs a mean smoothing on line transect data. The user specifies the cell size. The smoothed value in a cell is the mean of all observations in that cell and its surrounding eight nearest neighbouring cells. Each time the program is run, data from an individual cruises are read into `d`. This matrix `d` contains Easting, Northing and nutrient concentrations in the form

```
Easting, Northing, CHLa, NH4, NO2, PO4, SiO4,
Easting, Northing, CHLa, NH4, NO2, PO4, SiO4,
  ...  ,    ...  , ... , ..., ..., ..., ... ,
Easting, Northing, CHLa, NH4, NO2, PO4, SiO4.
```

The following output files are calculated: `xx0`, the Easting coordinates of the regular grid cells, `yy0`, the Northing coordinates of the regular grid cells, `smooth`, the smoothed values, `smoothe`, the number of observations used in each cell and `smoothd`, the standard deviation for each cell.

After smoothing, the files `smooth`, `smoothe` and `smoothd` need to be renamed in order not to be overwritten when the smoothing procedure is run for another cruise. We rename the files according to the cruise number; `smooth` was renamed to `m××`, `smoothd` to `m××d` and `smoothe` to `m××e`, where ×× are two digits indicating the cruise number. For example, `m09` is the smoothed values for Cruise 9.

```
/* define extremes of Bay */

minx=265000;maxx=337500;miny=5749000;maxy=5815500;
cls;
video("black","red","bold");     /* controls display on the
monitor */

frame(2,30,4,51,2);              /* draws a frame */

video("black","magenta","bold");
locate 3,32;"Spatial Smoothing";
video("black","cyan","bold");
locate 5,12;"This program performs a median smoothing on non-
gridded";
locate 6,12;"spatial data. User specifies 'box' dimensions dx and
dy.";
```

```
locate 7,12;"Data in a box are replaced by median of all
observations";
locate 8,12;"falling in that box AND those in neighbouring 3x3
boxes."
video("white","black","off");
locate 10,12;"Input requirements";
video("black","green","bold");
locate 12,12;"User specifies: (i)  GAUSS data file d.fmt
containing {x,y,Z1,...,Z6} data";
locate 13,12;"                (ii) box dimensions dx and dy";
video("white","black","off");
locate 15,12;"Program output";
video("black","green","bold");
locate 17,12;"                {xg,yg,Z1g,...,Z6g}";
locate 18,12;"     where     xg and yg are coordinates of box
centre";
locate 19,12;"      and      Zig is mean smoothed values for that
box.";
locate 20,1;"Output files are smooth.fmt—smoothed values,”;
locate 21,1;"                 smoothe.fmt-number of
observations,”;
locate 22,1;"                 smoothd.fmt-standard deviation,”;
locate 23,1;"                 xx.fmt-the first coordinator,”;
locate 24,1;"                 yy.fmt-the second coordinator.”;
video("black","yellow","blink");
locate 26,30;
dos pause;
video("black","white","off");
video("white","black","off");
locate 10,5;"GRID DETAILS";
video("black","white","bold");
locate 12,5;"Enter dx: ";;
video("black","yellow","bold");
dx=con(1,1);  /* input the first dimension of regular cells */

video("black","white","bold");
locate 13,5;"Enter dy: ";;
video("black","yellow","bold");
dy=con(1,1);     /* input the second dimension of regular cells
*/

loadm d;                           /* load the raw data */
x=d[.,1];y=d[.,2];z=d[.,3:ncol];

          /* compute the number of cells in the first coordinate
*/
nx=ceil((maxx-minx)/dx);

          /* compute the number of cells in the second coordinate
*/
ny=ceil((maxy-miny)/dy);
k=ncol-2;                          /* number of phenomena */
x0=minx;
y0=miny;
xgrid=seqa(x0+dx/2,dx,nx);
ygrid=seqa(y0+dy/2,dy,ny);

          /* create the vector of the first coordinate */

xx=xgrid.*ones(nx,ny);xx=vec(xx);

          /* create the vector of the second coordinate */

yy=ygrid’.*ones(nx,ny);yy=vec(yy);
smooth=zeros(k*nx,ny);smooth=miss(smooth,0);
smoothe=smooth;
```

```
smoothd=smooth;

          /* find the starting cell */

i=floor((minc(x)-x0)/dx)-4;
if i<0;
i=1;
endif;
cls;
format /ld 3,0;
video("black","green","bold");
locate 5,5;"Smoothing grid point ix=";
locate 5,35;"of";;nx;
locate 5,45;"iy=";
locate 5,49;"of";;ny;
format /ld 4,0;
video("black","yellow","bold");
do while i<=nx;
locate 5,29;i;
j=1;
do while j<=ny;
locate 5,43;j;

                 /* select samples */
e=ident(x,y,x0+dx*(2*i-1)/2,y0+dy*(2*j-1)/2,dx,dy);
if sumc(e)==0;
goto jump;
endif;
zz=selif(z,e);
rr=1;
do while rr<=k;
tempz=zz[.,rr]; tempz=packr(tempz);

               /* stack smoothed Z data vertically in blocks */
smooth[(rr-1)*nx+i,j]=meanc(tempz);smoothd[(rr-
1)*nx+i,j]=stdc(tempz);
smoothe[(rr-1)*nx+i,j]=rows(tempz);
if ismiss(meanc(tempz));
smoothe[(rr-1)*nx+i,j]=0;
endif;
rr=rr+1;
endo;
jump:
j=j+1;
endo;
tempe=x.<x0+dx*i;
x=delif(x,tempe);y=delif(y,tempe);z=delif(z,tempe);
i=i+1;
endo;
format /ld 9,4;
save smooth smoothd smoothe xx yy;

proc ident(x,y,c,r,dx,dy);

    /*     identifies elements of data matrix that have x,y
coordinates
           in current search area                   */

local e;
e=zeros(rows(x),1);
e=x.>=(c-1.5*dx) .and x.<=(c+1.5*dx) .and y.>=(r-1.5*dy) .and
y.<=(r+1.5*dy);
retp(e);
endp;
```

(C2) Masking Routine

The following procedure assigns a 0/1 value to variable `e` depending whether the elements of the coordinate vector {`xx0,yy0`} lies within or outside the Bay. `hd` contains the vertices of the boundary polygon in a clockwise direction. Notice that the boundary polygon is complicated, making direct computation very slow. To make the computations more efficient, we create a exterior polygon, `pext`, and a interior polygon, `pint`. Then assign a value of zero to cells outside the exterior polygon and a value of one to cells inside the interior polygon. Finally cells between the interior and exterior polygons are tested against the actual boundary polygon and assigned 0/1 accordingly.

```
e=ppbinout(xx0,yy0);           /* call the ppbinout routine */

proc ppbinout(x,y);
local n,direct,e,s1,s2,temp,pint,pext,hd;
load pext;                     /* load exterior polygon */
load pint;                     /* load interior polygon */
loadm hd;                      /* load boundary polygon */
n=rows(x);
if n<=10;
retp(inout(hd[.,1],hd[.,2],x,y));
endif;
cls;
locate 5,10;
video("black","cyan","bold");
"First Pass (bay interior):";

            /* mask the cells inside the internal polygon */

e=inout(pint[.,1],pint[.,2],x,y);
s1=selif(seqa(1,1,rows(e)),e);

            /* remove the cells inside the internal polygon */

temp=delif(x~y,e);
s2=delif(seqa(1,1,rows(e)),e);
locate 7,10;
"Second Pass (bay exterior):";

       /* mask the remaining cells inside the external polygon */

e=inout(pext[.,1],pext[.,2],temp[.,1],temp[.,2],direct);

       /* remove the cells which are outside the external polygon
*/

s2=selif(s2,e);
temp=selif(temp,e);
locate 9,10;
"Third Pass (resolving marginal points):";

 /* mask the marginal cells which are inside the boundary polygon
*/

e=inout(hd[.,1],hd[.,2],temp[.,1],temp[.,2]);
s2=selif(s2,e);
e=zeros(n,1);
temp=union(s1,s2,1);
```

```
e[temp,1]=ones(rows(temp),1);
retp(e);
endp;

  /*  The following procedure takes an ordered polygon (as
      specified by sequence of points in px and py) and
      assigns a 0/1 variable to variable IND depending on
      whether or not current point {xi,yi} lies outside or
      inside the polygon respectively. The routine can handle
      concave polygons.

      This procedure will used together with
                POLYCHK - determines if individual point
                          lies in a single convex polygon.

      For each {xi,yi} pair determine polygon line
      segments intercepted by vertical line at x=xi.        */

proc (1)=inout(px,py,x,y);
local e,e1,e2,ei,xi,yi,xd,yd;
e=zeros(rows(x),1);
xd=px~lag1(px);
xd=xd[2:rows(xd),.];     /* get the first coord. of each segment
*/
yd=py~lag1(py);
yd=yd[2:rows(yd),.];     /* get the second coord. of each segment
*/
i=0;
do while i<rows(x);
cls;
locate 10,5;"Checking";;i;;"of";;rows(x);
e1=((x[i,1].<xd[.,1]) .and (x[i,1].>xd[.,2]));
e2=((x[i,1].>xd[.,1]) .and (x[i,1].<xd[.,2]));
ei=e1+e2;
xi=selif(xd,ei);
yi=selif(yd,ei);
ei=sumc(ei);
e[i,1]=0;
if ei>0;
ei=polychk(x[i,1],y[i,1],xi,yi);
e[i,1]=(sumc(ei)<0);
endif;
endo;
retp(e);
endp;

  /*   The following procedure tests to see
       if point {x,y} lies inside the polygon. */

proc polychk(xi,yi,xd,yd);
local e,f,r,i;
r=rows(xd);e=zeros(r,1);
i=0;
do while i<r;
i=i+1;
f=(xd[i,1]-xd[i,2])*(yi-yd[i,1])-(yd[i,1]-yd[i,2])*(xi-xd[i,1]);
if f>0; e[i,1]=1;
elseif f<0; e[i,1]=-1; endif;
endo;
retp(e);
endp;
```

To determine where a grid cell lies, we assign a 0/1 value to variable `ei` (`i`=1, 2,⋯, 6) on the dases of whether the element of the coordinate vector {`xx0,yy0`} lies inside or outside Environmental Zone `i`. Assume that each environmental zone (`zone1` to `zone7`) was determined by its vertexes and given in a clockwise direction. The following program should be run.

```
load zone1 zone2 zone3 zone4 zone5 zone6 zone7;
      /* load polygon vector for each environmental zone. */
e1=inout(zone1[.,1],zone1[.,2],xx0,yy0);
e2=inout(zone2[.,1],zone2[.,2],xx0,yy0);
e3=inout(zone3[.,1],zone3[.,2],xx0,yy0);
e4=inout(zone4[.,1],zone4[.,2],xx0,yy0);
e5=inout(zone5[.,1],zone5[.,2],xx0,yy0);
e6=inout(zone6[.,1],zone6[.,2],xx0,yy0);
e7=inout(zone7[.,1],zone7[.,2],xx0,yy0);
```

(C3) Interpolation Procedure

The following procedure will be used by the nutrient distribution interpolator in (C5) and (C6). It performs an examination of the regular grid cells which are within the Bay. If there is no missing value, no action is taken. If a missing value is encountered we interpolate the value by using existing mean smoothed values. The interpolation method is the Completely Regularised Spline (CRS) with tension given by Mitasova, H. and Mitas, L., (1993). The user should specify: `dx`, `dy`, the first and second dimensions of the regular cell, `x`, `y`, first and second coordinates of all regular cells, `z`, the smoothed values in all regular cells, `e`, the indicator of the regular cell's locations, `mincell`, the minimum number of cells required for interpolation, and `psi`, the tension parameter.

```
proc (1)=inteplat(x,y,z,e,dx,dy,mincell,psi);

  /* x--the first coordinates,
     y--the second coordinates,
     e--indicator of whether an interpolation is needed
        1(yes), 0(no)
     dx--increment of x,
     dy--increment of y,
     mincell—minimum cells to be used for interpolation,
     z--observations,
     psi--tension parameter

    Related procedures:
                   SEGMENT - selects smoothed values to be used;
                   CRS - performs completely regularised spline

*/
local zhat,i,n,sx,sy,sz,
if rows(packr(z))<mincell;
```

```
cls;
locate 5,1; "The minmum number is too large.";
locate 7,1; "The number should not greater than";
locate 7,36; rows(packr(z));
retp(0);
endif;
n=rows(z);
zhat=z;
i=1;
do while i<n;
if (ismiss(zhat[i,1]) and e[i,1]==1);
{sx,sy,sz}=segment (x[i,1],y[i,1],x,y,z,dx,dy,mincell);
zhat[i,1]=crs(x[i,1],y[i,1],sx,sy,sz,psi);

          /* assign a zero value to negative result */

if (zhat[i,1]<0);
zhat[i,1]=0;
endif;
zhat[i,1];
endif;
i=i+1;
endo;
retp(zhat);
endp;

   /* This procedure collects the observations which will be used
to
   compute the interpolation of a given cell, by using the
algorithm
   for segmented processing suggested on page 646 of Mitasova,H.
and
   Mitas, L., (1993), Interpolation by Regularized Spline with
   tension,Mathematical Geology Vol. 25, No. 6: 641--655.
*/

proc (3)=segment(ix,iy,x,y,z,dx,dy,mincell);

  /* NOTE: x,y,z are all unpacked vectors.
     i--the index of a given cell,
     x--the first coordinate of all observations,
     y--the second coordinate of all observations,
     z--value of studied phenomena,
     dx--increment of x values,
     dy--increment of y values,
     mincell--minimum number of cells required for segmented
              processing.          */

local minx,maxx,miny,maxy;
local k,nn,temp,temp1;
temp=packr(x~y~z);
k=1;
nn=0;
do while nn<mincell;
k=k+1;
minx=ix-(k+0.1)*dx;
maxx=ix+(k+0.1)*dx;
miny=iy-(k+0.1)*dy;
maxy=iy+(k+0.1)*dy;
temp1=selif(temp,temp[.,1].>minx .and temp[.,1].<maxx);
if rows(packr(temp1))>2;
temp1=selif(temp1,temp1[.,2].>miny .and temp1[.,2].<maxy);
endif;
nn=rows(temp1);
endo;
retp(temp1[.,1],temp1[.,2],temp1[.,3]);
```

```
endp;

Proc (1)=crs(ix,iy,cx,cy,cz,psi);

  /* (ix,iy)--cell at which the phenomenon is estimated by CRS,
     cx--the first coordinates of the segmented data,
     cy--the second coordinate of the segmented data,
     cz--value of studied phenomena,
     psi-tesion parameter.                              */

local cn,cb,cc,czhat;
local ci,cj,ck,theta,ind;
cls;
locate 5,10;
print "Calculating weights ... Please wait";
cn=rows(cz);                     /* calculate observations */
              /* start assigning B matrix */
cb=zeros(cn,cn);
ind=zeros(cn,1);
ci=1;
do while ci<=cn-1;
cj=ci+1;
do while cj<=cn;
cb[ci,cj]=r(cx[ci,1],cy[ci,1],cx[cj,1],cy[cj,1],psi);
cb[cj,ci]=cb[ci,cj];
cj=cj+1;
endo;
ci=ci+1;
endo;
      /* Make sure there are no duplicate points in the data set
*/
ci=1;
do while ci<=cn;
cj=ci+1;
do while cj<=cn;
if cb[ci,cj]==0;
ind[cj,1]=1;
endif;
cj=cj+1;
endo;
ci=ci+1;
endo;
cb=delif(cb,ind);cb=cb';cb=delif(cb,ind);cb=cb';
cx=delif(cx,ind);cy=delif(cy,ind);cz=delif(cz,ind);
cn=cn-sumc(ind);
cb=cb|ones(1,cn);
cb=(ones(cn,1)|0)~cb;
cc=cz|0;
theta=cc/cb;
                    /* Interpolation routine */
format /lz;
czhat=theta[1,1];
ck=1;
do while ck<=cn;
czhat=czhat+theta[ck+1,1]*r(ix,iy,cx[ck,1],cy[ck,1],psi);
ck=ck+1;
endo;
format /ld 12,8;
cls;
locate 7,1; "Interpolation of (";;
ix;;",";;iy;;")=";
locate 8,10; czhat;
retp(czhat);
endp;
                 /* Calculate Radial basis function */
proc r(rxi,ryi,rxj,ryj,psi);
```

```
local delta,rij,rr,rz;
rij=sqrt((rxi-rxj)^2+(ryi-ryj)^2);
if rij==0;
retp(0);
endif;
rz=(psi*rij/2)^2;
if rz<=1;
rr=(-0.99999193+0.24991055*rz-0.05519968*rz^2)*rz;
rr=rr+(0.00976004-0.00107857*rz)*rz^4;
elseif rz>709;
rr=-ln(rz)-0.5772156649;
else;
rr=(rz^2+8.5733287401*rz+18.0590169730)*rz^2;
rr=rr+8.6347608925*rz+.2677737343;
delta=(rz^2+9.5733223454*rz+25.6329561486)*rz^2;
delta=delta+21.0996530827*rz+3.9584969228;
rr=-rr/delta/rz/exp(rz)-ln(rz)-0.5772156649;
endif;
retp(rr);
endp;
```

(C4) Combination Procedure

This procedure computes the annual and overall smoothed values, using monthly results. The yearly smoothed values were saved as `year1sm` and `year2sm`. Their standard errors were saved as `year1err` and `year2err`. The overall average and its standard error were saved as `yearsm` and `yearerr`.

```
k=rows(m01);l=cols(m01);

      /* assign matrix with missing values  */

year1sm=zeros(k,l);year1sm=miss(year1sm1,0);
year1err=year1sm;
year2sm=year1sm;
year2err=year1sm;
yearsm=year1sm;
yearerr=year1sm;

      /* start combination  */

do while i<k;
do while j<l;
tt1=0;nn1=0;ss1=0; /* initialize the counters for the first year
*/
tt2=0;nn2=0;ss2=0; /* initialize the counters for the second year
*/
tt=0;nn=0;ss=0;    /* initialize the counters for all cruises */


    /* add the mean s.e. and no. of observations into the
counters
```

```
      if the no. of observations is greater than 10
*/

if m01e[i,j]>10;
tt1=tt1+m01[i,j];nn1=nn1+1;
ss1=ss1+m01d[i,j]^2/m01e[i,j];
endif;

if m02e[i,j]>1;
tt1=tt1+m02[i,j];nn1=nn1+1;
ss1=ss1+m02d[i,j]^2/m02e[i,j];
endif;
if m03e[i,j]>1;
tt1=tt1+m03[i,j];nn1=nn1+1;
ss1=ss1+m03d[i,j]^2/m03e[i,j];
endif;
if m04e[i,j]>1;
tt1=tt1+m04[i,j];nn1=nn1+1;
ss1=ss1+m04d[i,j]^2/m04e[i,j];
endif;
if m05e[i,j]>1;
tt1=tt1+m05[i,j];nn1=nn1+1;
ss1=ss1+m05d[i,j]^2/m05e[i,j];
endif;
if m06e[i,j]>1;
tt1=tt1+m06[i,j];nn1=nn1+1;
ss1=ss1+m06d[i,j]^2/m06e[i,j];
endif;
if m07e[i,j]>1;
tt1=tt1+m07[i,j];nn1=nn1+1;
ss1=ss1+m07d[i,j]^2/m07e[i,j];
endif;
if m08e[i,j]>1;
tt1=tt1+m08[i,j];nn1=nn1+1;
ss1=ss1+m08d[i,j]^2/m08e[i,j];
endif;
if m09e[i,j]>1;
tt1=tt1+m09[i,j];nn1=nn1+1;
ss1=ss1+m09d[i,j]^2/m09e[i,j];
endif;
if m10e[i,j]>1;
tt1=tt1+m10[i,j];nn1=nn1+1;
ss1=ss1+m10d[i,j]^2/m10e[i,j];
endif;
if m11e[i,j]>1;
tt1=tt1+m11[i,j];nn1=nn1+1;
ss1=ss1+m11d[i,j]^2/m11e[i,j];
endif;
if m12e[i,j]>1;
tt1=tt1+m12[i,j];nn1=nn1+1;
ss1=ss1+m12d[i,j]^2/m12e[i,j];
endif;
if m13e[i,j]>1;
tt1=tt1+m13[i,j];nn1=nn1+1;
ss1=ss1+m13d[i,j]^2/m13e[i,j];
endif;

      /* calculation for the first year */

if nn1>0;
tt=tt1;nn=nn1;ss=ss1;       /* keep the values of the first year
*/
year1sm[i,j]=tt1/nn1;
year1err[i,j]=sqrt(ss1)/nn1;
endif;
locate 10,5; i;;j;
```

```
locate 11,5; year1sm [i,j];;year1err[i,j];

    /* start the calculation for the second year */

if m14e[i,j]>1;
tt2=tt2+m14[i,j];nn2=nn2+1;
ss2=ss2+m14d[i,j]^2/m14e[i,j];
endif;
if m15e[i,j]>1;
tt2=tt2+m15[i,j];nn2=nn2+1;
ss2=ss2+m15d[i,j]^2/m15e[i,j];
endif;
if m16e[i,j]>1;
tt2=tt2+m16[i,j];nn2=nn2+1;
ss2=ss2+m16d[i,j]^2/m16e[i,j];
endif;
if m17e[i,j]>1;
tt2=tt2+m17[i,j];nn2=nn2+1;
ss2=ss2+m17d[i,j]^2/m17e[i,j];
endif;
if m18e[i,j]>1;
tt2=tt2+m18[i,j];nn2=nn2+1;
ss2=ss2+m18d[i,j]^2/m18e[i,j];
endif;
if m19e[i,j]>1;
tt2=tt2+m19[i,j];nn2=nn2+1;
ss2=ss2+m19d[i,j]^2/m19e[i,j];
endif;
if m20e[i,j]>1;
tt2=tt2+m20[i,j];nn2=nn2+1;
ss2=ss2+m20d[i,j]^2/m20e[i,j];
endif;
if m21e[i,j]>1;
tt2=tt2+m21[i,j];nn2=nn2+1;
ss2=ss2+m21d[i,j]^2/m21e[i,j];
endif;
if m22e[i,j]>1;
tt2=tt2+m22[i,j];nn2=nn2+1;
ss2=ss2+m22d[i,j]^2/m22e[i,j];
endif;
if m23e[i,j]>1;
tt2=tt2+m23[i,j];nn2=nn2+1;
ss2=ss2+m23d[i,j]^2/m23e[i,j];
endif;
if m24e[i,j]>1;
tt2=tt2+m24[i,j];nn2=nn2+1;
ss2=ss2+m24d[i,j]^2/m24e[i,j];
endif;
if m25e[i,j]>1;
tt2=tt2+m25[i,j];nn2=nn2+1;
ss2=ss2+m25d[i,j]^2/m25e[i,j];
endif;
if nn2>0;
tt=tt+tt2;nn=nn+nn2;ss=ss+ss2;
/* combine the values for all cruises */

        /* calculation for the second year */

year2sm[i,j]=tt2/nn2;
year2err[i,j]=sqrt(ss2)/nn2;
endif;
locate 12,5; i;;j;
locate 13,5; year2sm[i,j];;year2err[i,j];

    /* calculation for all cruises */
```

```
if nn>0;
yearsm[i,j]=tt/nn;
yearerr[i,j]=sqrt(ss)/nn;
endif;
locate 14,5; i;;j;
locate 15,5; yearsm[i,j];;yearerr[i,j];
endo;
endo;
```

(C5) Interpolating Annual Distributions

The following program performs interpolation of the nutrient distributions for the first year, using the interpolation procedure given in Appendix C3. Easting and Northing need to be rescaled before interpolation starts because original numbers are too large to compute the radial basis. Output files were named by `y1,` followed by nutrient names. For example, `y1chla` stores the interpolation result of CHLa in the first year. To compute interpolation results in the second year, simply replace `year1sm` by `year2sm` and `y1`×××× by `y2`×××× in the program and rerun. The distributions of standard errors of the annual means, overall average and its standard error can be computed in the similar way.

```
dx=0.05;dy=dx;psi=70;mincell=100;
xx=(xx0-minc(xx0))/10000;yy=(yy0-minc(yy0))/10000;
            /* Interpolation for CHLa */
zz=year1sm[1:145,.];     /* getting smoothed values from stacked
                            matrix */
zz=vec(zz);              /* reshape to a vector */
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1chla=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation
*/
save y1chla;             /* save the interpolation result */
y1chla=0;                /* free the worksapce */

                 /* Interpolation for NH4 */

zz=year1sm[146:290,.];   /* getting smoothed values from stacked
                            matrix */
zz=vec(zz);              /* reshape to a vector */
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1nh4=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation */
save y1nh4;              /* save the interpolation result */
y1nh4=0;                 /* free the worksapce */

                /* Interpolation for NO2 */

zz=year1sm[291:435,.];   /* getting smoothed values from stacked
                            matrix */
zz=vec(zz);              /* reshape to a vector */
```

```
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1no2=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation */
save y1no2;              /* save the interpolation result */
y1no2=0;                 /* free the worksapce */

              /* Interpolation for PO4 */

zz=year1sm[436:580,.];   /* getting smoothed values from stacked
                            matrix */
zz=vec(zz                /* reshape to a vector */
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1po4=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation */
save y1po4;              /* save the interpolation result */
y1po4=0;                 /* free the worksapce */

                /* Interpolation for NO3 */

zz=year1sm[581:725,.];   /* getting smoothed values from stacked
                            matrix */
zz=vec(zz);              /* reshape to a vector */
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1no3=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation */
save y1no3;              /* save the interpolation result */
y1no3=0;                 /* free the worksapce */

           /* Interpolation for SiO4 */

zz=year1sm[726:870,.];   /* getting smoothed values from stacked
                            matrix */
zz=vec(zz);              /* reshape to a vector */
zz=missex(zz,e.==0);     /* setting the values outside the Bay to
                            missing */
y1sio4=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation
*/
save y1sio4;             /* save the interpolation result */
y1sio4=0;                /* free the worksapce */
```

(C6) Interpolating Monthly Distributions

The following program performs the interpolation on each nutrient for the first month. The program to perform the interpolation for the other months can be written in a similar way. We assume that the interpolation results for the overall average were saved under the name of `ov` followed by nutrient names. For example, `ovchla` denote the interpolation result for CHLa. The predetermined distance is 5km.

```
 /* define the values for grid size, tension and minimum No. used
*/

dx=0.05;dy=dx;psi=70;mincell=100;
```

```
mindist=0.5;

            /* Interpolation for CHLa */
zz=m01[1:145,.];nn=m01e[1:145,.];  /*getting the values from
stacked
                                     matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
zz=missex(zz,e.==0);
modi=modifica(zz,ovchla);       /* calculate the amount of
adjustment */

zz=fhole(xx,yy,zz,mindist,modi,ovchla);   /* fill the gaps */
chla10=intepl1(xx,yy,zz,e,dx,dy,mincell,psi);  /* Interpolation
*/
save chla01;                         /* save the result */
chla01=0;                            /* free the workspace */

              /* Interpolation for NH4 */

zz=m01[146:290,.];nn=m01e[146:290,.]; /*getting the values from
                                        stacked matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
zz=missex(zz,e.==0);
modi=modifica(zz,ovnh4);    /* calculate the amount of adjustment
*/
zz=fhole(xx,yy,zz,mindist,modi,ovnh4); /* fill the gaps */
nh401=intepl1(xx,yy,zz,e,dx,dy,mincell,psi); /* Interpolation */
save nh401;                  /* save the result */
nh401=0;                     /* free the workspace */

               /* Interpolation for NO2 */

zz=m01[291:435,.];nn=m01e[291:435,.]; /*getting the values from
                                        stacked matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
zz=missex(zz,e.==0);
modi=modifica(zz,ovno2);    /* calculate the amount of adjustment
*/
zz=fhole(xx,yy,zz,mindist,modi,ovno2); /* fill the gaps */
no201=intepl1(xx,yy,zz,e,dx,dy,mincell,psi); /* Interpolation */
save no201;                 /* save the result */
no201=0;                    /* free the workspace */

                /* Interpolation for PO4 */

zz=m01[436:580,.];nn=m01e[436:580,.]; /*getting the values from
                                        stacked matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
zz=missex(zz,e.==0);
modi=modifica(zz,ovpo4);    /* calculate the amount of adjustment
*/
zz=fhole(xx,yy,zz,mindist,modi,ovnh4); /* fill the gaps */
po401=intepl1(xx,yy,zz,e,dx,dy,mincell,psi); /* Interpolation */
save po401;                  /* save the result */
po401=0;                     /* free the workspace */

                /* Interpolation for NO3 */

zz=m01[581:725,.];nn=m01e[581:725,.]; /*getting the values from
                                        stacked matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
```

```
zz=missex(zz,e.==0);
modi=modifica(zz,ovno3);    /* calculate the amount of adjustment
*/
zz=fhole(xx,yy,zz,mindist,modi,ovno3); /* fill the gaps */
no301=intepl1(xx,yy,zz,e,dx,dy,mincell,psi); /* Interpolation */
save no301;                  /* save the result */
no301=0;                     /* free the workspace */

                /* Interpolation for SiO4 */

zz=m01[726:870,.];nn=m01e[726:870,.]; /*getting the values from
                                        stacked matrix */
zz=vec(zz);nn=vec(nn);
zz=missex(zz,nn.<10);
zz=missex(zz,e.==0);
modi=modifica(zz,ovsio4);   /* calculate the amount of adjustment
*/
zz=fhole(xx,yy,zz,mindist,modi,ovsio4); /* fill the gaps */
sio401=intepl1(xx,yy,zz,e,dx,dy,mincell,psi); /* Interpolation */
save sio401;                  /* save the result */
sio401=0;                     /* free the workspace */

/* Calculate the amount of adjustment for each environmental zone
*/

proc modifica(zz,zz_all);
local moditerm modi0,modi;
modi=zeros(7,1);
moditerm=zz~zz_all;
modi0=selif(moditerm,e1);
modi[1,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 1*/
modi0=packr(modi0);
if modi0>0;
            /* calculate the mean difference in Zone 1*/

modi[1,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;modi0=selif(moditerm,e2);

modi[2,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 2*/

modi0=packr(modi0);
if modi0>0;
            /* calculate the mean difference in Zone 2*/

modi[2,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
modi0=selif(moditerm,e3);
modi[3,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 3*/

modi0=packr(modi0);
if modi0>0;
              /* calculate the mean difference in Zone 3*/

modi[3,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
modi0=selif(moditerm,e4);
modi[4,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 4*/
```

```
modi0=packr(modi0);
if modi0>0;
         /* calculate the mean difference in Zone 4*/

modi[4,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
modi0=selif(moditerm,e5);
modi[5,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 5*/

modi0=packr(modi0);
if modi0>0;
           /* calculate the mean difference in Zone 5*/

modi[5,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
modi0=selif(moditerm,e6);
modi[6,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 6*/

modi0=packr(modi0);
if modi0>0;
                 /* calculate the mean difference in Zone 6*/

modi[6,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
modi0=selif(moditerm,e7);
modi[7,1]=sumc(modi0[.,2])/rows(modi0[.,2]); /* calculate the
mean
                                                value in Zone 7*/

modi0=packr(modi0);
if modi0>0;
              /* calculate the mean difference in Zone 7*/

modi[7,1]=(sumc(modi0[.,1])-sumc(modi0[.,2]))/rows(modi0);
endif;
retp(modi);
endp;

/* Insert a value to the cell which is more than far away from
the sample points, The minimum distance is specified by mindist.
*/

proc (1)=fhole(xx,yy,zz, mindist,modi,almean);
local t,r,i,zz1,ind,d;
zz1=zz;d=mindist^2;
t=xx~yy~zz;
t=packr(t);
t=t[.,1:2];
r=rows(xx);
i=0;
do while i<r;
i=i+1;
cls;i;
if (ismiss(zz1[i,1]) and e[i,1]==1);
        /* check the distance from sample locations */
ind=selif(t,((t[.,1]-xx[i,1])^2.+(t[.,2]-yy[i,1])^2).<d);
ind=packr(ind);ind=sumc(ind);
if ismiss(ind);

    /* assign a value according to the zones */

if e1[i,1]==1;
```

```
zz1[i,1]=almean[i,1]+modi[1,1];
elseif e2[i,1]==1;
zz1[i,1]=almean[i,1]+modi[2,1];
elseif e3[i,1]==1;
zz1[i,1]=almean[i,1]+modi[i,1];
elseif e4[i,1]==1;
zz1[i,1]=almean[i,1]+modi[4,1];
elseif e5[i,1]==1;
zz1[i,1]=almean[i,1]+modi[5,1];
elseif e6[i,1]==1;
zz1[i,1]=almean[i,1]+modi[6,1];
elseif e7[i,1]==1;
zz1[i,1]=almean[i,1]+modi[7,1];
endif;
if zz1[i,1]<0; zz1[i,1]=0; endif;
xx[i,1];;yy[i,1];;zz1[i,1];
cc;
endif;
endif;
endo;
retp(zz1);
endp;
```

(C7) Boundary Determination

The following procedure is used to select the coordinates of the boundary cells and save them under the name `boundary`.

```
boundary=bound(xx0,yy0,e,500,500); /* call boundary procedure */

     /* the following procedure select the cells which have a
        common boundary or point with interior cells
        x- the first coord. of the regular cells
        y- the second coord. of the regular cells
        z- the indicators which indicate the interior cells
        dx,dy- the cell size */

proc (1)=bound(x,y,e,dx,dy);
local boundarye1,e2,e3,e4,e5,e6,e7,e8,ee;
boundary=x~y;  /* combine the coord. */
boundary=delif(boundary,e);    /* remove the interior cells */
e1=ppbinout(boundary[.,1]-dx,boundary[.,2]);
/* check the west side */
e2=ppbinout(boundary[.,1]+dx,boundary[.,2]);
/* check the east side */
e3=ppbinout(boundary[.,1],boundary[.,2]-dy);
/* check the south side */
e4=ppbinout(boundary[.,1],boundary[.,2]+dy);
/* check the north side */
e5=ppbinout(boundary[.,1]-dx,boundary[.,2]-dy);
/* check south-west corner */
e6=ppbinout(boundary[.,1]+dx,boundary[.,2]-dy);
/* check south-east corner */
e7=ppbinout(boundary[.,1]-dx,boundary[.,2]+dy);
/* check north-west corner */
e8=ppbinout(boundary[.,1]+dx,boundary[.,2]+dy);
/* check north-east corner */
ee=e1+e2+e3+e4+e5+e6+e7+e8;
```

```
boundary=selif(boundary,ee);
retp(boundary);
endp;
```

(C8) Interpolating Bathymetric Distribution

The following program interpolates the bathymetry distribution, using predefined sounding samples and assign sound reading on the boundary cells to be zero. We assume the sounding samples were stored in the file `sound.fmt`. The output file is saved in `depth.fmt`.

```
     /* assign the values for tension ect. */
dx=0.05;dy=dx;psi=45;mincell=80;

    /* combine the sample locations, boundary and regular cells
*/

xx=sound[.,1]|boundary[.,1]|xx0;
yy=sound[.,2]|boundary[.,2]|yy0;

    /* rescale the coord. */

xx=(xx-minc(xx))/10000;yy=(yy-minc(yy))/10000;
zz=zeros(rows(e),1);zz=miss(zz,0);
zz=sound[.,3]|zeros(rows(boundary,1)|zz; /* combine the depth */

depth=inteplat(xx,yy,zz,e,dx,dy,mincell,psi); /* interpolation */

/* select the depth for the regular cells */

depth=depth[rows(depth)-rows(e):rows(depth),1];
save depth;
```

APPENDIX D

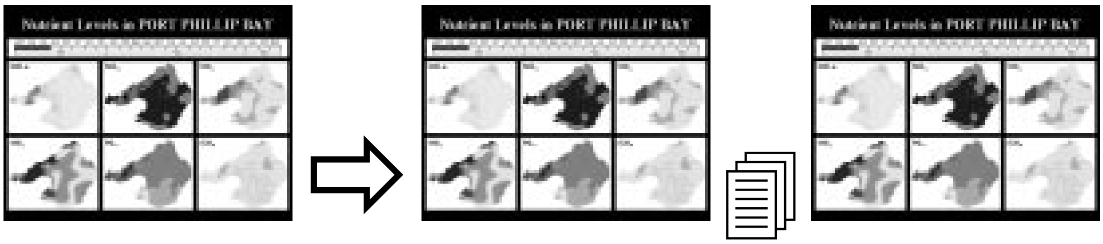
COMPUTER ANIMATIONS:

TECHNICAL DESCRIPTION AND PLAYING INSTRUCTIONS

(D1) Technical details

In this appendix we describe how individual static maps of interpolated nutrient concentrations were combined into a single animated video sequence.

The original SAS® images in BMP format were “cropped”, “resized” and converted to TGA format. Six nutrient maps, one for each cruise, were combined onto a standard black background in predetermined positions. To obtain a smooth animation between frames, three additional “morphed” frames were generated to slot between each pair of cruise frames within the time sequence.



The morphed images were combined (in sequence) into a single video file. This animation file can then be viewed by using the MS-Windows video player.

The last process, combining the animation file with an appropriate player to form a standalone executable program introduced the undesirable feature of lines appearing within the time sequence. Therefore, the animation file (PPBVIDEO.FLC) was used and not the standalone executable file.

(D2) Playing Instruction

To view animation, enter a DOS window, change to directory [CD ROM Driver]:\ppbvideo and type PPBVIDEO, or double click the file PPBVIDEO.BAT under file manager of Window 3.1 (or Windows 95 windows explorer or later). This uses the freely distributable program PlayFLC Version 3.10, however this player requires a VESA driver to have been loaded as a TSR (Terminal Stay Resident) in computer memory. Pressing the numerical keys during playing will change the speed. Pressing [Esc] key will exit playing.

To view individual nutrients, run any multi-media player that will accept an *.AVI file, such as MPLAYER in Windows for workgroups 3.11 in the ACCESSORIES group.

10. TABLES

Table 1. Estimated annual nutrient pool sizes (tonnes)

(data in brackets give the upper limit of 95% confidence interval)

Nutrient	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
				Year 1, May 1993—April 1994 (Cruise 1 to 13)				
CHLa	1.860 (1.875)	1.156 (1.165)	1.961 (1.982)	1.999 (2.030)	1.252 (1.263)	13.975 (14.103)	0.207 (0.209)	**22.401 (22.628)**
NH4	14.483 (14.697)	9.524 (9.661)	11.224 (11.529)	36.138 (37.440)	9.588 (9.736)	119.327 (121.629)	1.481 (1.544)	**201.766 (206.235)**
NO2	1.212 (1.256)	0.726 (0.751)	1.167 (1.227)	2.541 (2.628)	1.146 (1.173)	9.979 (10.377)	0.149 (0.158)	**16.920 (17.570)**
NO3	4.702 (4.832)	1.736 (1.799)	8.547 (8.888)	11.642 (12.049)	4.181 (4.268)	25.814 (26.653)	2.847 (3.031)	**59.469 (61.520)**
PO4	88.163 (88.711)	83.539 (83.823)	86.232 (86.537)	132.196 (133.368)	94.400 (94.699)	1022.202 (1026.216)	5.347 (5.391)	**1512.078 (1518.744)**
SiO4	244.505 (245.957)	271.349 (272.127)	284.887 (286.490)	257.957 (259.263)	176.770 (177.456)	2856.820 (2866.354)	43.803 (45.095)	**4136.091 (4152.742)**
				Year 2, May 1994—March 1995 (Cruise 14 to 25)				
CHLa	1.616 (1.629)	1.106 (1.113)	1.368 (1.382)	1.634 (1.655)	1.398 (1.413)	11.754 (11.832)	0.198 (0.200)	**19.073 (19.224)**
NH4	8.641 (8.865)	4.911 (4.997)	6.102 (6.203)	17.750 (18.182)	8.818 (9.006)	71.257 (72.613)	0.802 (0.822)	**118.282 (120.687)**
NO2	1.278 (1.376)	0.636 (0.655)	0.812 (0.832)	1.563 (1.600)	0.786 (0.809)	6.842 (7.093)	0.118 (0.121)	**12.036 (12.486)**
NO3	3.161 (3.336)	2.068 (2.128)	5.337 (5.550)	9.238 (9.452)	4.640 (4.724)	20.977 (21.687)	1.645 (1.734)	**47.066 (48.611)**
PO4	78.301 (78.849)	81.265 (81.493)	87.631 (87.926)	118.190 (118.837)	90.824 (91.202)	1000.315 (1003.510)	6.025 (6.052)	**1462.552 (1467.870)**
SiO4	127.287 (128.079)	130.191 (130.746)	149.833 (150.510)	164.559 (165.202)	138.690 (139.224)	1658.258 (1664.064)	21.631 (21.858)	**2390.449 (2399.683)**

Table 2. Estimated Average nutrient pool sizes (tonnes) over all cruises

(data in brackets give the upper limit of 95% confidence interval)

Nutrient	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
CHLa	1.860 (1.871)	1.154 (1.161)	1.691 (1.707)	1.807 (1.829)	1.347 (1.358)	13.295 (13.391)	0.195 (0.198)	**21.349 (21.515)**
NH4	12.472 (12.571)	6.299 (6.348)	9.330 (9.429)	32.509 (33.001)	9.941 (10.033)	96.171 (97.128)	1.145 (1.167)	**167.867 (169.677)**
NO2	1.114 (1.134)	0.666 (0.676)	0.992 (1.010)	2.319 (2.351)	0.924 (0.935)	7.940 (8.081)	0.135 (0.137)	**14.090 (14.322)**
NO3	4.003 (4.016)	1.934 (1.964)	7.057 (7.173)	12.027 (12.181)	4.618 (4.657)	24.345 (24.734)	2.484 (2.529)	**56.468 (57.300)**
PO4	86.251 (86.465)	81.818 (81.936)	86.269 (86.403)	126.857 (127.318)	92.100 (92.270)	1015.509 (1017.111)	5.828 (5.842)	**1494.632 (1497.344)**
SiO4	188.212 (190.444)	168.848 (169.943)	212.315 (214.214)	209.877 (211.820)	146.818 (147.819)	2068.160 (2181.045)	31.863 (32.874)	**3026.092 (3048.158)**

Table 3. Estimated Bay-wide Chlorophyll 'a' pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
1	25-26/05/93	2.160	1.434	1.476	1.518	1.309	17.297	0.128	**25.321**
		(2.328)*	(1.553)	(1.579)	(1.632)	(1.389)	(18.369)	(0.141)	**(26.992)**
2	22-	2.920	1.905	1.208	1.559	1.139	22.593	0.124	**31.448**
	25/06/93	(3.382)	(2.196)	(1.370)	(1.781)	(1.230)	(25.714)	(0.151)	**(35.824)**
3	20-	2.443	1.088	1.099	3.660	2.043	16.672	0.087	**27.090**
	21/07/93	(2.637)	(1.196)	(1.285)	(4.002)	(2.190)	(18.326)	(0.097)	**(29.732)**
4	03-	2.560	1.121	1.585	2.595	1.866	14.280	0.127	**24.135**
	04/08/93	(2.872)	(1.285)	(1.766)	(2.959)	(2.102)	(16.172)	(0.151)	**(27.306)**
5	14-	2.118	1.087	3.281	3.168	1.607	14.499	0.226	**25.986**
	15/09/93	(2.293)	(1.213)	(3.738)	(3.761)	(1.931)	(15.976)	(0.259)	**(29.171)**
6	12-	1.880	1.361	4.455	1.413	1.073	14.374	0.262	**24.818**
	13/10/93	(2.049)	(1.514)	(5.186)	(1.589)	(1.176)	(16.027)	(0.293)	**(27.835)**
7	09-	1.651	1.314	5.154	6.117	1.357	14.949	0.399	**30.941**
	11/11/93	(1.788)	(1.448)	(5.929)	(7.282)	(1.454)	(16.455)	(0.495)	**(34.851)**
8	07-	0.932	0.791	1.268	0.954	0.724	8.533	0.149	**13.350**
	08/12/93	(1.009)	(0.846)	(1.360)	(1.046)	(0.758)	(9.108)	(0.171)	**(14.298)**
9	04-	2.129	1.810	3.571	1.870	1.380	18.463	0.328	**29.552**
	07/01/94	(2.215)	(1.949)	(3.936)	(2.056)	(1.443)	(19.435)	(0.372)	**(31.407)**
10	01-	1.499	0.990	1.753	1.711	1.172	12.299	0.209	**19.634**
	03/02/94	(1.747)	(1.116)	(1.897)	(2.174)	(1.425)	(14.064)	(0.227)	**(22.650)**
11	01-	1.974	1.171	2.259	2.583	1.430	14.422	0.215	**24.053**
	03/03/94	(2.136)	(1.234)	(2.524)	(2.988)	(1.505)	(15.303)	(0.244)	**(25.936)**
12	28-	1.068	0.616	0.899	1.236	0.762	6.141	0.180	**10.903**
	30/03/94	(1.142)	(0.657)	(1.015)	(1.392)	(0.801)	(6.466)	(0.200)	**(11.672)**
13	26-	1.747	1.167	1.279	1.717	1.174	13.930	0.154	**21.170**
	29/04/94	(1.885)	(1.254)	(1.354)	(1.871)	(1.249)	(14.980)	(0.184)	**(22.778)**
14	31/05-	3.666	1.906	1.340	2.358	2.107	23.560	0.080	**35.017**
	03/06/94	(3.844)	(1.971)	(1.452)	(2.488)	(2.226)	(24.899)	(0.100)	**(36.979)**
15	22,27-	1.888	1.127	1.278	1.564	1.083	14.172	0.095	**21.207**
	29/06/94	(1.970)	(1.178)	(1.335)	(1.674)	(1.129)	(14.786)	(0.103)	**(22.175)**
16	19-	1.674	1.090	1.159	2.925	1.878	12.007	0.078	**20.812**
	21/07/94	(1.881)	(1.220)	(1.384)	(3.645)	(2.204)	(13.389)	(0.091)	**(23.812)**
17	16-	1.311	1.014	0.733	1.223	1.273	9.061	0.053	**14.667**
	18/08/94	(1.403)	(1.104)	(0.823)	(1.388)	(1.374)	(9.805)	(0.057)	**(15.955)**
18	14-	1.684	1.383	1.096	3.699	1.351	11.450	0.118	**20.782**
	16/09/94	(1.852)	(1.551)	(1.344)	(4.317)	(1.501)	(12.934)	(0.137)	**(23.636)**
19	12-	0.906	0.739	1.608	0.601	0.658	7.173	0.129	**11.813**
	14/10/94	(0.980)	(0.791)	(1.892)	(0.669)	(0.695)	(7.707)	(0.140)	**(12.873)**
20	09-	1.473	1.406	2.099	0.881	0.862	9.655	0.289	**16.665**
	11/11/94	(1.574)	(1.594)	(2.449)	(1.066)	(0.923)	(10.464)	(0.334)	**(18.404)**
21	06-	1.103	0.572	1.209	1.261	0.892	7.143	0.129	**12.308**
	10/12/94	(1.207)	(0.630)	(1.452)	(1.464)	(1.010)	(7.876)	(0.163)	**(13.802)**
22	03-	1.318	0.614	0.734	1.231	0.883	7.669	0.089	**12.537**
	05/01/95	(1.403)	(0.641)	(0.832)	(1.337)	(0.924)	(8.144)	(0.105)	**(13.386)**
23	31/01-	1.791	1.119	1.598	1.404	1.425	11.626	0.373	**19.336**
	02/02/95	(1.922)	(1.235)	(1.788)	(1.644)	(1.596)	(12.535)	(0.488)	**(21.211)**
24	27/02-	1.762	1.443	1.699	1.699	1.982	15.721	0.213	**24.517**
	02/03/95	(1.925)	(1.565)	(1.879)	(1.855)	(2.258)	(16.970)	(0.256)	**(26.707)**
25	27-	1.868	1.247	1.592	1.119	1.405	17.349	0.169	**24.748**
	29/03/95	(2.040)	(1.384)	(1.821)	(1.244)	(1.574)	(19.048)	(0.194)	**(27.305)**

Table 4 Estimated Bay-wide Ammonium 'NH4' pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
1	25-26/05/93	14.317 (16.893)	9.878 (11.645)	8.620 (10.986)	25.479 (33.715)	7.588 (8.653)	110.500 (128.450)	0.824 (1.139)	**177.205 (211.281)**
2	22- 25/06/93	23.394 (29.999)	13.023 (16.061)	10.700 (14.940)	49.393 (66.697)	16.261 (19.127)	209.983 (272.659)	1.441 (2.499)	**324.196 (421.983)**
3	20- 21/07/93	15.805 (19.227)	11.969 (14.066)	14.314 (19.345)	140.374 (168.542)	37.182 (41.523)	148.655 (173.870)	2.540 (3.226)	**370.838 (439.798)**
4	03- 04/08/93	17.658 (20.245)	9.551 (11.600)	5.745 (7.284)	38.020 (46.505)	27.052 (31.167)	139.442 (162.892)	0.368 (0.564)	**237.835 (280.257)**
5	14- 15/09/93	12.975 (15.673)	6.641 (8.372)	61.368 (77.621)	136.041 (174.582)	7.153 (8.580)	156.817 (193.601)	1.419 (2.401)	**382.416 (480.829)**
6	12- 13/10/93	13.219 (15.038)	9.384 (10.669)	9.087 (11.781)	24.983 (31.238)	9.826 (11.472)	136.273 (153.685)	0.405 (0.809)	**203.176 (234.692)**
7	09- 11/11/93	12.571 (13.682)	9.376 (10.345)	11.311 (13.653)	36.965 (42.955)	7.104 (8.093)	122.079 (134.869)	1.733 (2.001)	**201.138 (225.598)**
8	07- 08/12/93	18.246 (20.338)	11.022 (12.892)	7.944 (9.878)	10.550 (12.752)	8.242 (9.483)	147.086 (170.935)	1.141 (1.286)	**204.231 (237.563)**
9	04- 07/01/94	11.396 (12.783)	6.008 (6.933)	5.464 (6.222)	11.528 (14.021)	6.822 (7.561)	93.511 (103.493)	1.388 (1.894)	**136.117 (152.908)**
10	01- 03/02/94	12.476 (13.743)	5.666 (6.432)	7.483 (8.521)	7.176 (8.090)	7.073 (7.759)	89.343 (99.858)	1.411 (2.017)	**130.628 (146.420)**
11	01- 03/03/94	11.319 (13.203)	6.692 (7.509)	9.810 (12.222)	6.769 (9.084)	5.368 (6.003)	94.003 (105.535)	1.723 (2.353)	**135.683 (155.908)**
12	28- 30/03/94	10.849 (12.089)	6.088 (6.801)	4.146 (4.843)	18.797 (19.858)	5.590 (6.176)	93.696 (104.720)	0.364 (0.430)	**139.530 (154.917)**
13	26- 29/04/94	11.633 (14.681)	5.942 (8.048)	4.318 (5.983)	7.110 (9.448)	7.145 (8.563)	94.404 (119.549)	0.817 (1.109)	**131.370 (167.381)**
14	31/05- 03/06/94	10.864 (12.776)	5.497 (6.684)	6.635 (8.419)	16.405 (21.796)	8.491 (9.564)	63.901 (79.444)	1.452 (1.950)	**113.245 (140.633)**
15	22,27- 29/06/94	13.978 (16.630)	5.521 (6.545)	6.460 (7.523)	58.655 (71.756)	30.130 (36.273)	88.191 (102.662)	0.622 (0.726)	**203.556 (242.116)**
16	19- 21/07/94	12.496 (17.474)	4.195 (5.937)	3.351 (5.807)	40.745 (56.439)	11.065 (13.719)	88.364 (118.078)	0.600 (8.805)	**160.815 (218.259)**
17	16- 18/08/94	11.854 (13.574)	7.434 (8.412)	6.779 (7.748)	28.993 (33.653)	6.068 (6.882)	93.317 (108.701)	0.803 (1.012)	**155.248 (179.984)**
18	14- 16/09/94	6.689 (7.772)	2.826 (3.618)	4.984 (5.554)	7.582 (8.283)	5.715 (6.357)	52.153 (61.088)	0.316 (0.355)	**80.265 (93.028)**
19	12- 14/10/94	9.691 (11.871)	5.017 (6.098)	7.560 (8.952)	35.266 (54.127)	4.862 (5.687)	91.163 (105.529)	1.053 (1.360)	**154.612 (193.625)**
20	09- 11/11/94	10.557 (11.709)	5.355 (6.160)	5.894 (7.363)	17.017 (22.573)	6.925 (7.637)	80.178 (90.753)	1.104 (1.543)	**127.030 (147.739)**
21	06- 10/12/94	8.751 (11.459)	3.602 (5.213)	4.648 (5.704)	10.438 (11.646)	5.565 (7.130)	71.402 (92.063)	0.352 (0.421)	**104.758 (133.635)**
22	03- 05/01/95	5.536 (7.313)	4.949 (6.226)	3.214 (4.338)	12.770 (14.810)	6.077 (7.248)	63.601 (83.331)	0.100 (0.153)	**96.247 (123.419)**
23	31/01- 02/02/95	7.815 (9.103)	4.910 (5.653)	6.610 (8.253)	6.987 (8.655)	5.063 (6.324)	116.622 (135.653)	1.224 (1.453)	**149.231 (175.094)**
24	27/02- 02/03/95	7.203 (9.190)	3.633 (4.865)	5.000 (5.744)	6.688 (7.847)	2.996 (4.205)	62.623 (75.196)	0.626 (0.800)	**88.767 (107.846)**
25	27- 29/03/95	8.723 (10.067)	3.700 (4.561)	5.322 (6.299)	6.453 (7.162)	3.909 (4.515)	57.608 (68.093)	0.514 (0.649)	**86.229 (101.346)**

Table 5 Estimated Bay-wide Nitrite 'NO2' pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Total
1	25-26/05/93	1.058 (1.355)	0.327 (0.469)	0.500 (0.739)	3.652 (4.528)	0.466 (0.668)	5.414 (7.606)	0.021 (0.038)	**11.438 (15.403)**
2	22- 25/06/93	1.660 (2.318)	1.181 (1.560)	1.609 (2.250)	3.184 (4.386)	1.068 (1.386)	18.876 (24.497)	0.181 (0.355)	**27.759 (36.754)**
3	20- 21/07/93	1.296 (1.652)	0.706 (0.913)	1.128 (1.804)	5.131 (5.997)	2.434 (2.680)	8.500 (11.460)	0.220 (0.289)	**19.415 (24.796)**
4	03- 04/08/93	2.369 (3.005)	1.472 (1.905)	0.940 (1.347)	7.962 (9.097)	5.205 (5.844)	17.104 (22.422)	0.054 (0.085)	**35.106 (43.704)**
5	14- 15/09/93	1.835 (2.682)	0.521 (0.861)	5.327 (8.574)	6.889 (9.322)	0.544 (0.841)	12.023 (18.346)	0.170 (0.542)	**27.310 (41.168)**
6	12- 13/10/93	1.049 (1.445)	1.081 (1.330)	2.222 (2.858)	1.886 (2.563)	0.482 (0.673)	12.265 (15.211)	0.071 (0.132)	**19.055 (24.212)**
7	09- 11/11/93	5.789 (0.849)	0.585 (0.659)	0.414 (0.697)	0.428 (0.912)	0.725 (0.442)	0.292 (6.004)	3.294 (0.104)	**0.052 (9.657)**
8	07- 08/12/93	0.771 (1.176)	0.585 (0.858)	0.720 (1.035)	0.494 (0.842)	0.320 (0.511)	6.945 (10.289)	0.051 (0.088)	**9.886 (14.798)**
9	04- 07/01/94	1.525 (1.978)	0.860 (1.128)	0.860 (1.116)	1.482 (2.054)	0.671 (0.875)	10.869 (14.060)	0.226 (0.297)	**16.495 (21.508)**
10	01- 03/02/94	1.567 (2.008)	0.654 (0.883)	0.610 (0.857)	2.614 (3.072)	1.032 (1.246)	11.312 (14.381)	0.107 (0.162)	**17.896 (22.609)**
11	01- 03/03/94	1.389 (1.781)	1.087 (1.409)	1.140 (1.572)	1.338 (2.025)	0.474 (0.665)	9.075 (12.416)	0.137 (0.240)	**14.640 (20.110)**
12	28- 30/03/94	0.566 (0.749)	0.510 (0.647)	0.406 (0.585)	0.514 (0.678)	0.389 (0.504)	5.755 (7.508)	0.042 (0.050)	**8.183 (10.720)**
13	26- 29/04/94	1.499 (2.180)	1.102 (1.679)	0.432 (0.769)	1.699 (2.300)	0.849 (1.167)	11.383 (17.444)	0.105 (0.132)	**17.069 (25.673)**
14	31/05- 03/06/94	2.306 (2.819)	0.831 (1.087)	1.117 (1.435)	3.410 (4.228)	1.184 (1.437)	8.978 (12.500)	0.185 (0.267)	**18.011 (23.772)**
15	22,27- 29/06/94	1.875 (2.573)	1.202 (1.495)	1.031 (1.284)	4.614 (5.473)	2.326 (2.873)	11.993 (15.634)	0.083 (0.102)	**23.123 (29.435)**
16	19- 21/07/94	1.926 (2.586)	0.526 (0.878)	1.162 (1.547)	3.785 (4.671)	1.687 (2.075)	10.726 (15.265)	0.108 (0.129)	**19.921 (27.151)**
17	16- 18/08/94	1.363 (1.796)	0.771 (1.050)	1.152 (1.450)	2.657 (3.197)	0.443 (0.623)	7.268 (10.715)	0.128 (0.154)	**13.781 (18.984)**
18	14- 16/09/94	0.783 (1.205)	0.376 (0.547)	0.206 (0.302)	0.698 (0.950)	0.383 (0.564)	4.366 (6.959)	0.046 (0.057)	**6.857 (10.583)**
19	12- 14/10/94	0.560 (0.784)	0.304 (0.410)	0.386 (0.678)	1.418 (2.201)	0.352 (0.524)	4.701 (6.766)	0.101 (0.153)	**7.822 (11.516)**
20	09- 11/11/94	0.507 (0.555)	0.842 (1.147)	1.645 (2.112)	1.004 (1.622)	0.327 (0.481)	6.342 (9.052)	0.308 (0.385)	**10.974 (15.653)**
21	06- 10/12/94	0.424 (0.619)	0.284 (0.437)	0.270 (0.448)	0.404 (0.542)	0.229 (0.303)	3.552 (5.484)	0.021 (0.028)	**5.184 (7.861)**
22	03- 05/01/95	0.560 (0.858)	0.306 (0.504)	0.620 (0.940)	1.398 (1.833)	0.750 (0.952)	5.275 (8.387)	0.048 (0.069)	**8.958 (13.542)**
23	31/01- 02/02/95	0.695 (0.948)	0.391 (0.571)	0.391 (0.559)	0.335 (0.445)	0.210 (0.345)	4.040 (6.070)	0.056 (0.109)	**6.118 (9.047)**
24	27/02- 02/03/95	1.890 (3.278)	0.428 (0.655)	0.860 (1.032)	0.616 (0.831)	0.494 (0.648)	5.205 (8.065)	0.148 (0.173)	**9.600 (14.683)**
25	27- 29/03/95	0.885 (1.150)	0.580 (0.783)	0.559 (0.708)	0.409 (0.538)	0.461 (0.596)	6.772 (9.186)	0.077 (0.099)	**9.743 (13.060)**

Table 6 Estimated Bay-wide Nitrate 'N03' pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
1	25-26/05/93	9.240 (10.786)	2.365 (3.210)	6.564 (9.683)	15.145 (19.068)	3.723 (4.503)	47.535 (60.488)	0.713 (1.228)	**85.284 (108.967)**
2	22- 25/06/93	8.982 (12.500)	4.926 (6.604)	5.446 (8.525)	12.492 (17.066)	4.263 (5.293)	67.443 (90.236)	1.006 (1.421)	**104.558 (141.644)**
3	20- 21/07/93	9.932 (11.755)	4.761 (5.752)	8.526 (13.498)	29.985 (34.735)	15.970 (17.413)	49.598 (62.278)	8.169 (10.066)	**126.941 (155.496)**
4	03- 04/08/93	7.086 (8.483)	4.022 (4.910)	3.621 (6.683)	28.471 (33.463)	22.371 (24.942)	46.258 (57.165)	1.528 (2.814)	**113.357 (138.461)**
5	14- 15/09/93	4.154 (5.326)	1.113 (1.887)	46.863 (62.505)	43.875 (60.224)	2.038 (2.677)	40.417 (59.654)	4.123 (7.942)	**142.584 (200.214)**
6	12- 13/10/93	2.111 (2.909)	0.942 (1.452)	16.217 (24.842)	6.640 (8.997)	1.537 (2.045)	15.483 (23.783)	1.905 (4.674)	**44.834 (68.702)**
7	09- 11/11/93	5.033 (5.687)	1.075 (1.558)	17.823 (23.778)	2.358 (2.974)	1.347 (1.601)	11.078 (16.200)	4.605 (5.530)	**43.320 (57.329)**
8	07- 08/12/93	2.289 (3.248)	0.902 (1.584)	2.974 (5.414)	1.159 (1.854)	1.117 (1.588)	8.988 (17.692)	0.152 (0.514)	**17.582 (31.893)**
9	04- 07/01/94	6.622 (7.325)	2.685 (3.080)	8.380 (10.948)	6.168 (7.273)	3.640 (3.988)	38.292 (43.037)	3.548 (7.345)	**69.335 (82.995)**
10	01- 03/02/94	2.206 (2.886)	0.931 (1.312)	2.444 (3.359)	4.204 (4.965)	2.500 (3.040)	13.187 (17.686)	1.514 (2.399)	**26.986 (35.647)**
11	01- 03/03/94	3.333 (4.306)	2.357 (2.819)	11.039 (14.237)	5.788 (8.288)	1.975 (2.371)	31.810 (36.749)	1.635 (3.282)	**57.935 (72.053)**
12	28- 30/03/94	3.161 (3.898)	2.261 (2.910)	2.201 (2.840)	1.819 (2.222)	1.484 (1.814)	19.695 (25.818)	0.135 (0.173)	**30.756 (39.675)**
13	26- 29/04/94	3.995 (5.438)	1.852 (2.736)	0.842 (1.587)	3.345 (4.543)	2.073 (3.049)	22.925 (33.219)	0.293 (0.405)	**35.326 (50.976)**
14	31/05- 03/06/94	2.891 (3.781)	0.676 (0.940)	3.323 (4.906)	15.433 (19.174)	5.778 (6.664)	6.891 (11.203)	1.756 (2.815)	**36.747 (49.483)**
15	22,27- 29/06/94	5.717 (7.427)	3.215 (4.016)	10.279 (15.573)	32.130 (38.517)	16.792 (19.000)	36.446 (47.936)	1.688 (2.179)	**106.267 (134.648)**
16	19- 21/07/94	5.898 (8.527)	0.994 (2.200)	6.370 (8.639)	25.812 (32.224)	11.050 (12.875)	16.703 (30.672)	0.505 (0.685)	**67.334 (95.823)**
17	16- 18/08/94	6.650 (8.537)	5.168 (6.739)	7.468 (9.942)	17.259 (20.404)	2.598 (3.344)	38.656 (51.825)	2.276 (2.854)	**80.074 (103.645)**
18	14- 16/09/94	1.337 (2.007)	0.689 (0.948)	2.476 (2.847)	4.127 (4.573)	1.221 (1.502)	6.019 (9.903)	2.026 (2.125)	**17.895 (23.906)**
19	12- 14/10/94	1.520 (2.108)	0.955 (1.326)	7.708 (11.878)	5.476 (8.134)	1.754 (2.223)	13.529 (19.284)	4.140 (7.310)	**35.456 (52.263)**
20	09- 11/11/94	2.601 (3.875)	2.484 (3.244)	10.089 (12.931)	4.932 (7.030)	1.611 (1.957)	25.448 (33.141)	2.443 (3.491)	**49.607 (65.669)**
21	06- 10/12/94	0.720 (1.328)	0.605 (1.083)	0.903 (1.134)	1.124 (1.508)	0.823 (1.161)	5.429 (10.374)	0.231 (0.252)	**9.835 (16.839)**
22	03- 05/01/95	1.908 (2.767)	0.901 (1.571)	1.431 (2.078)	2.783 (3.756)	2.080 (2.513)	17.173 (26.178)	0.100 (0.141)	**26.376 (39.005)**
23	31/01- 02/02/95	2.734 (3.503)	2.189 (2.738)	4.259 (6.389)	1.601 (1.812)	1.088 (1.305)	19.988 (24.880)	0.545 (1.094)	**32.404 (41.720)**
24	27/02- 02/03/95	3.370 (5.644)	1.811 (2.352)	3.094 (4.125)	2.963 (3.470)	1.596 (2.010)	22.399 (29.744)	0.600 (0.788)	**35.832 (48.133)**
25	27- 29/03/95	5.085 (6.390)	3.309 (4.000)	2.024 (2.281)	2.999 (3.148)	2.307 (2.710)	29.616 (37.296)	0.583 (0.602)	**45.923 (56.426)**

Table 7 Estimated Bay-wide Phosphate 'PO4' pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Total
1	25-26/05/93	102.316 (108.909)	109.818 (113.616)	102.942 (106.880)	174.534 (186.319)	105.058 (108.109)	1193.832 (1242.713)	6.463 (6.928)	**1794.963 (1873.473)**
2	22- 25/06/93	112.553 (122.226)	87.767 (95.721)	96.555 (103.073)	153.591 (173.524)	106.889 (110.517)	1191.648 (1283.941)	6.969 (7.643)	**1755.972 (1896.645)**
3	20- 21/07/93	103.139 (107.898)	90.480 (94.453)	90.443 (96.165)	171.891 (188.294)	111.203 (117.569)	1133.845 (1176.572)	7.319 (8.102)	**1708.320 (1789.053)**
4	03- 04/08/93	102.460 (108.652)	89.924 (93.984)	85.771 (89.211)	136.260 (147.332)	113.645 (118.888)	1128.860 (1177.616)	4.944 (5.194)	**1661.864 (1740.879)**
5	14- 15/09/93	105.344 (109.241)	88.150 (90.395)	55.419 (60.145)	92.129 (100.323)	98.340 (100.432)	1115.812 (1148.524)	4.199 (4.698)	**1559.393 (1613.757)**
6	12- 13/10/93	91.342 (96.087)	79.309 (82.055)	84.056 (88.887)	117.851 (124.469)	90.121 (93.384)	1002.517 (1038.952)	4.736 (4.965)	**1469.933 (1528.800)**
7	09- 11/11/93	84.752 (89.260)	75.373 (78.274)	57.538 (60.577)	107.901 (111.095)	87.049 (89.492)	943.764 (979.151)	2.937 (3.209)	**1359.315 (1411.057)**
8	07- 08/12/93	74.381 (77.449)	75.578 (77.496)	83.476 (86.848)	120.400 (126.459)	86.062 (87.794)	943.055 (970.612)	4.191 (4.375)	**1387.144 (1431.034)**
9	04- 07/01/94	71.237 (74.829)	74.498 (76.584)	82.551 (84.760)	116.829 (124.071)	80.835 (82.885)	945.539 (970.328)	5.134 (5.463)	**1376.624 (1418.920)**
10	01- 03/02/94	87.192 (95.490)	80.079 (83.285)	87.209 (90.121)	111.630 (117.820)	87.428 (90.097)	970.939 (1003.867)	5.518 (5.725)	**1429.996 (1486.405)**
11	01- 03/03/94	77.553 (85.565)	75.720 (77.852)	82.826 (85.298)	142.153 (152.494)	98.392 (101.214)	961.793 (986.717)	5.051 (5.393)	**1443.488 (1494.533)**
12	28- 30/03/94	93.691 (98.005)	87.394 (90.090)	86.199 (89.066)	121.475 (130.053)	87.030 (89.603)	1086.681 (1115.568)	5.822 (6.100)	**1568.292 (1618.485)**
13	26- 29/04/94	83.951 (91.521)	80.605 (84.565)	99.258 (102.452)	132.748 (142.855)	88.535 (92.032)	1011.684 (1068.131)	7.363 (7.812)	**1504.144 (1589.369)**
14	31/05- 03/06/94	86.842 (91.993)	77.090 (80.363)	75.580 (78.527)	141.089 (152.163)	98.063 (101.015)	990.663 (1029.459)	4.973 (5.437)	**1474.299 (1538.958)**
15	22,27- 29/06/94	106.701 (113.090)	77.232 (79.254)	73.962 (76.026)	154.689 (168.628)	111.045 (115.077)	959.110 (998.915)	4.497 (4.725)	**1487.237 (1555.715)**
16	19- 21/07/94	86.925 (95.803)	81.369 (85.404)	76.525 (82.164)	151.823 (165.276)	110.314 (115.021)	970.618 (1026.899)	4.373 (4.701)	**1481.946 (1575.268)**
17	16- 18/08/94	77.261 (87.399)	83.328 (87.050)	72.841 (78.127)	134.482 (141.770)	90.013 (92.454)	917.770 (959.371)	4.191 (4.642)	**1379.886 (1450.813)**
18	14- 16/09/94	66.797 (72.133)	70.986 (74.998)	89.913 (93.563)	133.123 (139.331)	84.033 (87.143)	912.063 (954.761)	5.536 (5.618)	**1362.450 (1427.547)**
19	12- 14/10/94	76.145 (81.661)	82.960 (86.921)	101.762 (106.992)	111.622 (116.930)	85.958 (88.787)	1051.248 (1097.251)	5.856 (6.066)	**1515.551 (1584.609)**
20	09- 11/11/94	77.434 (80.891)	85.397 (87.816)	95.868 (101.064)	100.681 (105.306)	79.621 (81.330)	1038.363 (1064.851)	8.145 (8.857)	**1485.508 (1530.116)**
21	06- 10/12/94	72.880 (77.014)	82.425 (86.143)	85.770 (89.800)	105.803 (110.064)	71.271 (74.633)	1033.287 (1074.080)	5.852 (6.201)	**1457.287 (1517.935)**
22	03- 05/01/95	74.863 (80.628)	78.869 (82.428)	85.776 (89.504)	97.982 (102.620)	81.257 (84.054)	992.141 (1036.975)	5.850 (6.110)	**1416.739 (1482.318)**
23	31/01- 02/02/95	76.789 (80.657)	82.762 (84.658)	85.755 (89.110)	112.443 (124.143)	75.232 (81.819)	1058.058 (1086.342)	5.923 (6.108)	**1496.961 (1552.836)**
24	27/02- 02/03/95	85.300 (90.508)	89.368 (92.550)	92.902 (94.919)	110.094 (116.376)	98.773 (105.716)	1002.952 (1035.179)	6.683 (6.879)	**1486.071 (1542.127)**
25	27- 29/03/95	78.646 (83.304)	84.514 (87.675)	101.174 (104.502)	112.060 (115.507)	85.938 (88.823)	1075.426 (1116.574)	6.345 (6.470)	**1544.104 (1602.855)**

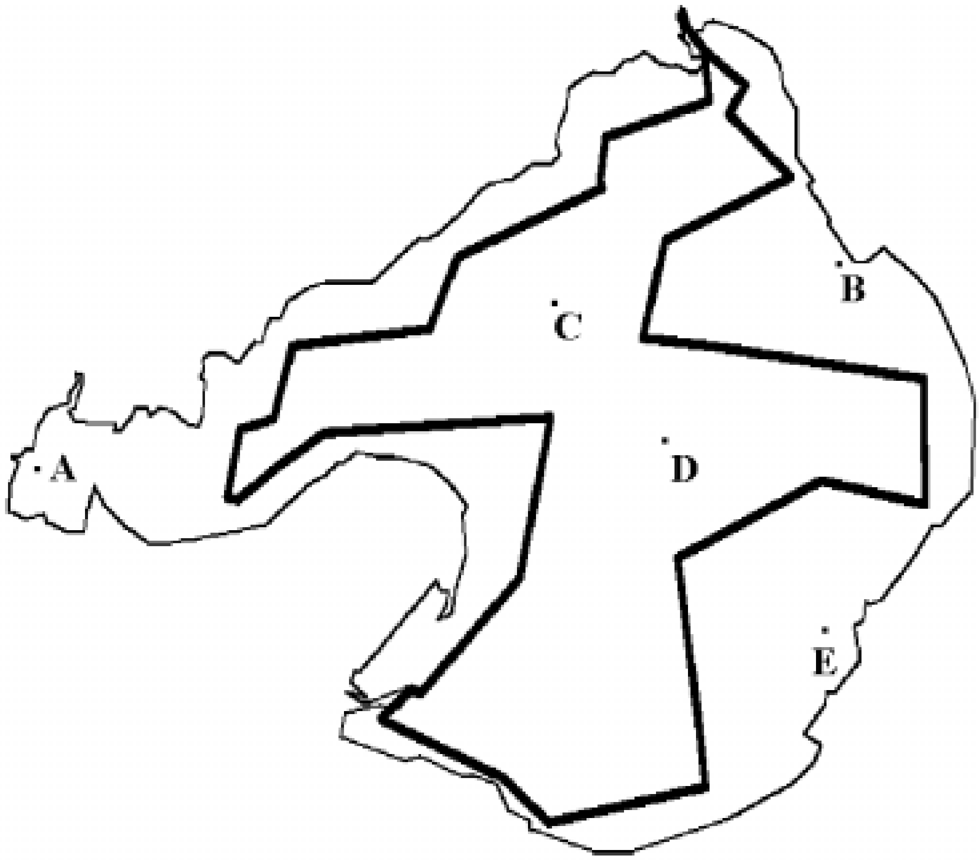
Table 8 Estimated Bay-wide Silicate ‘SiO4’ pool size (tonnes) by cruise

(data in brackets give the upper limit of 95% confidence interval)

Cruise	Date	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	**Total**
1	25-26/05/93	398.639 (418.306)	395.782 (407.551)	417.548 (428.666)	276.513 (292.424)	217.992 (225.965)	4501.289 (4631.689)	27.264 (28.909)	**6235.026 (6433.508)**
2	22-25/06/93	400.357 (474.842)	316.852 (339.087)	327.128 (369.707)	235.625 (280.954)	162.806 (173.687)	3986.985 (4287.490)	51.398 (104.789)	**5481.151 (6030.555)**
3	20-21/07/93	171.150 (190.474)	213.431 (225.289)	234.232 (272.224)	203.706 (230.179)	152.697 (162.743)	2279.744 (2435.000)	57.560 (71.014)	**3312.521 (3586.922)**
4	03-04/08/93	58.360 (75.226)	123.296 (136.204)	128.113 (146.679)	135.206 (149.949)	103.661 (115.691)	1149.230 (1286.581)	13.659 (17.329)	**1711.524 (1927.658)**
5	14-15/09/93	171.153 (203.505)	107.050 (124.071)	283.190 (351.787)	412.654 (473.467)	161.823 (194.056)	1463.339 (1713.795)	38.986 (72.419)	**2638.195 (3133.099)**
6	12-13/10/93	67.031 (84.664)	50.246 (64.042)	207.561 (273.413)	44.020 (57.445)	62.485 (75.338)	620.593 (748.112)	14.471 (32.580)	**1066.408 (1335.594)**
7	09-11/11/93	197.963 (213.823)	212.104 (222.300)	386.196 (460.579)	226.619 (233.804)	154.132 (159.459)	2376.057 (2480.385)	51.255 (70.716)	**3604.326 (3841.066)**
8	07-08/12/93	261.524 (277.463)	252.074 (263.181)	313.740 (331.057)	255.173 (268.133)	164.573 (172.172)	3060.105 (3190.103)	21.883 (25.302)	**4329.072 (4527.412)**
9	04-07/01/94	289.522 (308.533)	296.804 (311.324)	372.100 (396.340)	333.716 (356.710)	229.931 (237.337)	3943.073 (4074.419)	54.735 (68.104)	**5519.882 (5752.767)**
10	01-03/02/94	262.692 (312.746)	230.077 (255.442)	260.078 (286.738)	392.457 (437.894)	228.856 (252.077)	3251.754 (3599.399)	25.661 (32.767)	**4651.576 (5177.063)**
11	01-03/03/94	188.212 (190.444)	168.848 (169.943)	212.315 (214.214)	209.877 (211.820)	146.817 (147.819)	2068.160 (2081.045)	31.863 (32.874)	**3026.092 (3048.158)**
12	28-30/03/94	154.726 (165.669)	166.483 (176.485)	110.196 (123.425)	111.593 (121.844)	111.674 (117.586)	1499.043 (1605.861)	11.294 (11.910)	**2165.009 (2322.780)**
13	26-29/04/94	364.468 (397.962)	339.900 (367.844)	383.126 (403.265)	332.895 (360.305)	246.039 (262.741)	3935.953 (4212.982)	25.955 (29.356)	**5628.336 (6034.455)**
14	31/05-03/06/94	174.996 (201.394)	214.315 (230.069)	221.697 (252.590)	217.430 (240.941)	169.497 (182.081)	2069.199 (2311.840)	36.266 (51.743)	**3103.401 (3470.658)**
15	22,27-29/06/94	65.140 (85.006)	38.647 (49.231)	71.772 (95.194)	135.617 (152.649)	190.434 (210.313)	522.818 (649.122)	10.073 (12.680)	**1034.501 (1254.193)**
16	19-21/07/94	77.985 (103.584)	23.673 (32.427)	63.888 (82.345)	242.307 (282.600)	186.349 (206.129)	552.367 (692.805)	3.347 (4.781)	**1149.916 (1404.671)**
17	16-18/08/94	81.433 (98.080)	99.882 (112.718)	70.185 (86.200)	54.709 (62.779)	50.132 (56.359)	938.885 (1054.323)	11.524 (17.352)	**1306.749 (1487.811)**
18	14-16/09/94	62.178 (72.744)	55.175 (65.352)	68.745 (76.835)	37.152 (43.590)	42.521 (47.797)	799.179 (910.187)	8.394 (10.132)	**1073.345 (1226.638)**
19	12-14/10/94	71.358 (86.734)	79.886 (88.841)	97.952 (124.029)	92.351 (104.403)	84.088 (89.865)	887.379 (994.269)	34.202 (49.275)	**1347.214 (1537.417)**
20	09-11/11/94	174.452 (188.687)	197.564 (210.057)	224.233 (244.646)	198.420 (209.686)	141.821 (148.095)	2209.502 (2324.071)	22.171 (28.435)	**3168.163 (3353.677)**
21	06-10/12/94	193.885 (209.258)	191.608 (203.833)	147.990 (162.699)	208.262 (224.114)	168.669 (177.467)	2412.608 (2545.831)	12.494 (15.272)	**3335.516 (3538.474)**
22	03-05/01/95	132.045 (153.267)	150.820 (165.395)	127.826 (143.505)	259.612 (282.014)	183.073 (195.195)	1983.582 (2181.500)	10.633 (12.094)	**2847.620 (3132.970)**
23	31/01-02/02/95	212.052 (222.787)	217.060 (231.772)	321.159 (335.515)	181.339 (192.188)	96.103 (108.483)	2864.612 (2970.828)	28.942 (31.907)	**3921.268 (4093.478)**
24	27/02-02/03/95	259.341 (274.219)	259.169 (277.141)	281.231 (289.648)	245.731 (256.002)	157.046 (166.045)	3026.982 (3130.112)	19.987 (21.282)	**4249.487 (4414.449)**
25	27-29/03/95	78.354 (87.846)	59.746 (67.333)	50.974 (61.205)	76.075 (83.728)	63.008 (69.691)	808.730 (892.963)	9.586 (11.729)	**1146.473 (1274.494)**

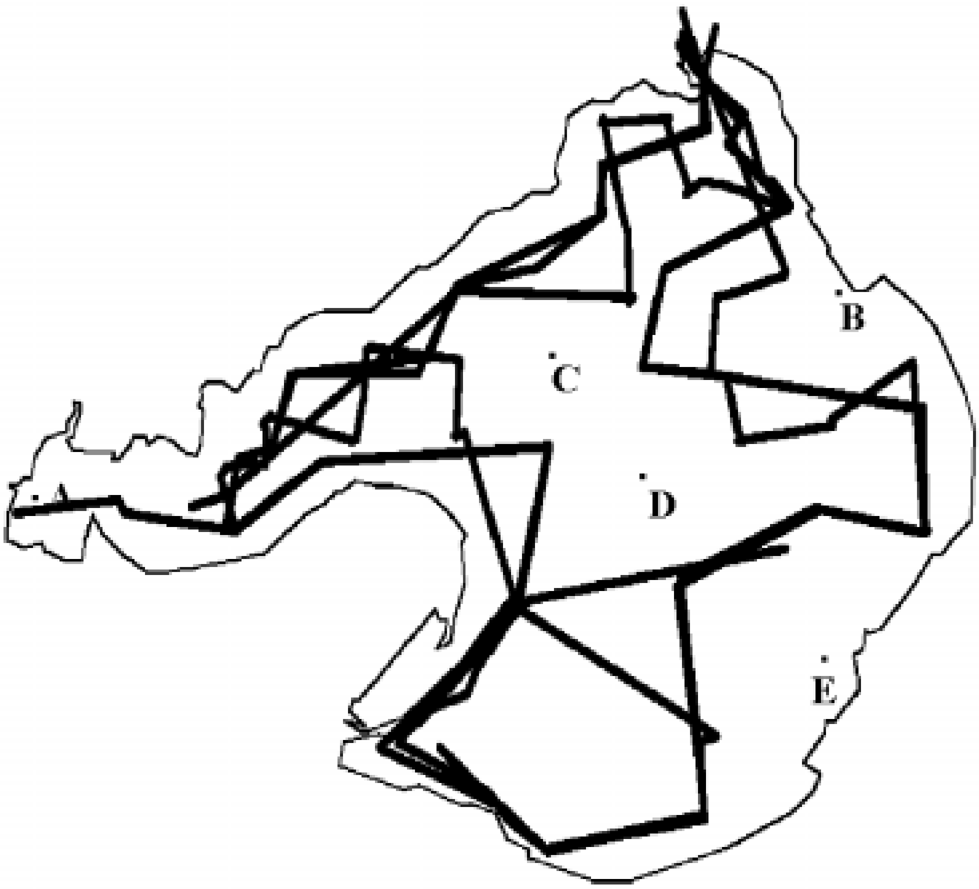
11. FIGURES

Figure 1. Path of typical cruise, May, 1993-March, 1995



Note that points A, B, E are more than 20km from the nearest transect line. Points C and D are more than 10km from the nearest transect line.

Figure 2. Plot of sample points for the first three cruises



Representative example of spatial coverage after pooling three cruises. Point E is still more than 20km from the nearest transect line. Point D is more than 10km from the nearest transect line.

Figure 3. Plot of sample points for the cruises sailed between May 1993 and April 1994

Spatial coverage obtained after pooling all cruises for one year (May 1993-April 1994). The maximum distance from any point to the nearest transect line is less than 5km.

The region around point O is very shallow and so the contribution to the total nutrient pools is small.

Figure 4. Locations of sounding data provided by RAN and MSL

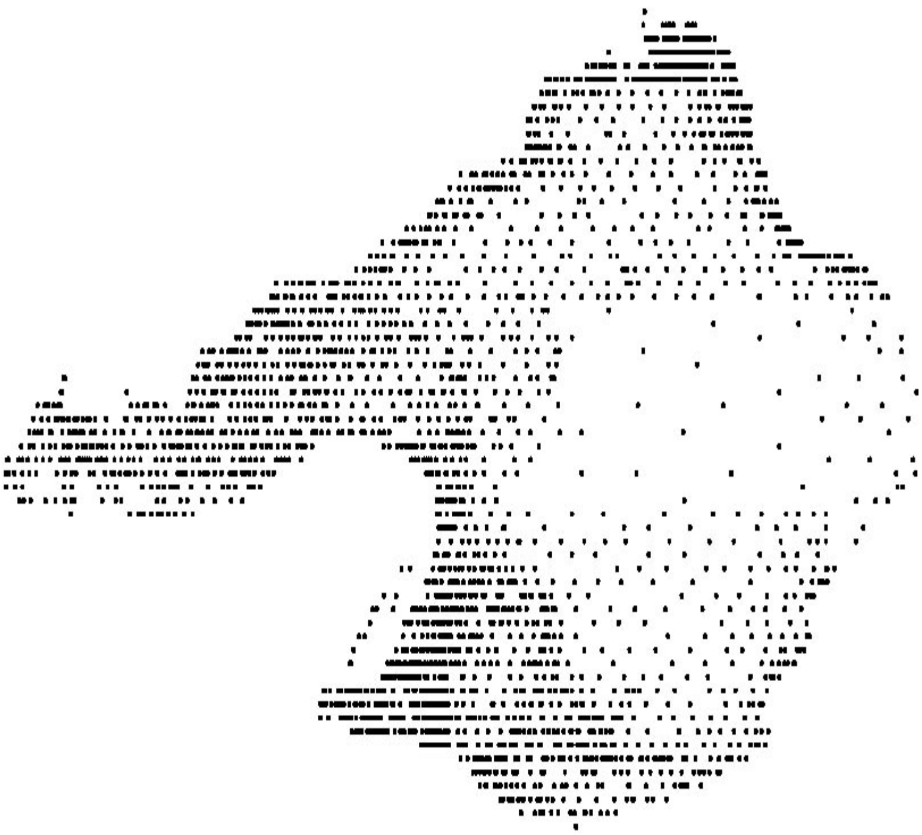


Figure 5. Enviromental zones in the Port Phillip Bay used in Phase One study

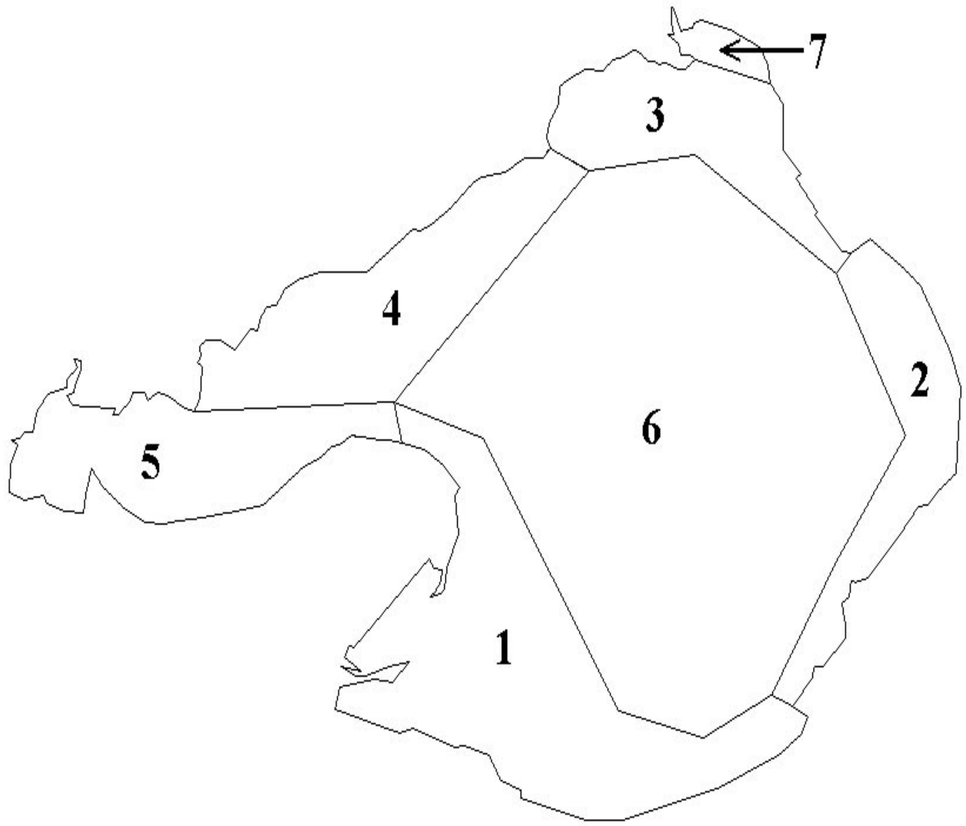
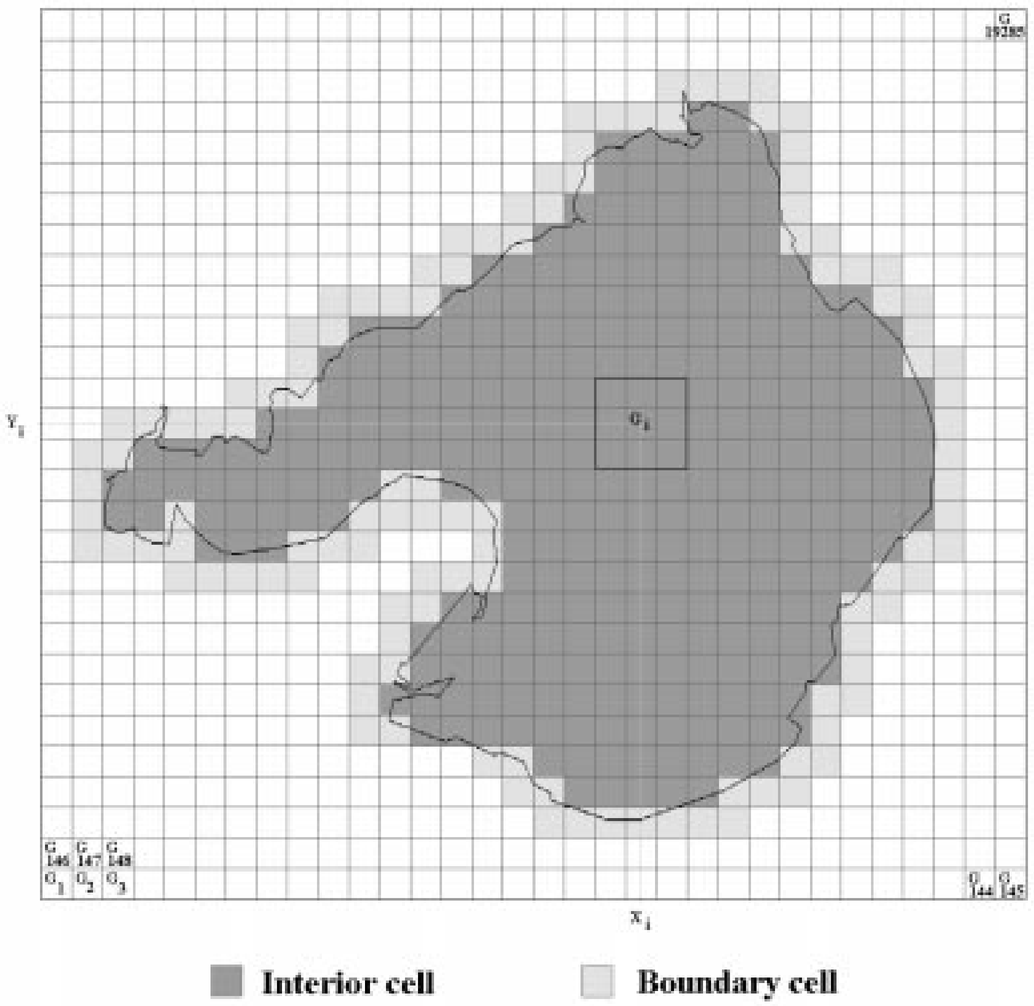
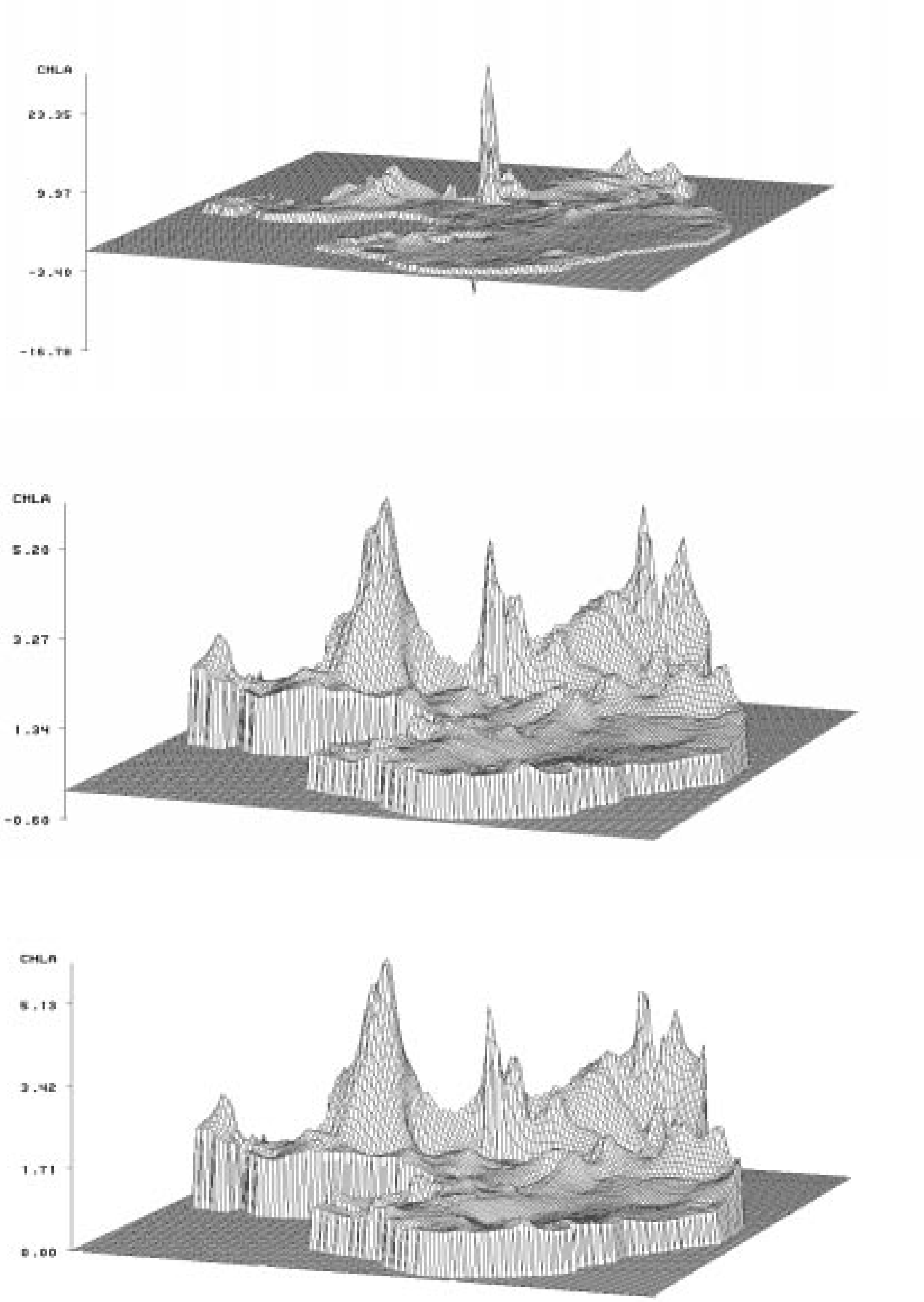


Figure 6. Illustration of partitioning of the Bay for smoothing and interpolation



$G_1, G_2, \cdots, G_i, \cdots$ indicate the regular cells.
(X_i, Y_i) is the centriod of cell G_i.

Figure 7. Illustration of effect of different tension on interpolation



From top to Bottom, tension φ =20, 40 and 100, respectively.
(Note different vertical scales).

Figure 8. Interpolated bathymetry map of Port Phillip Bay

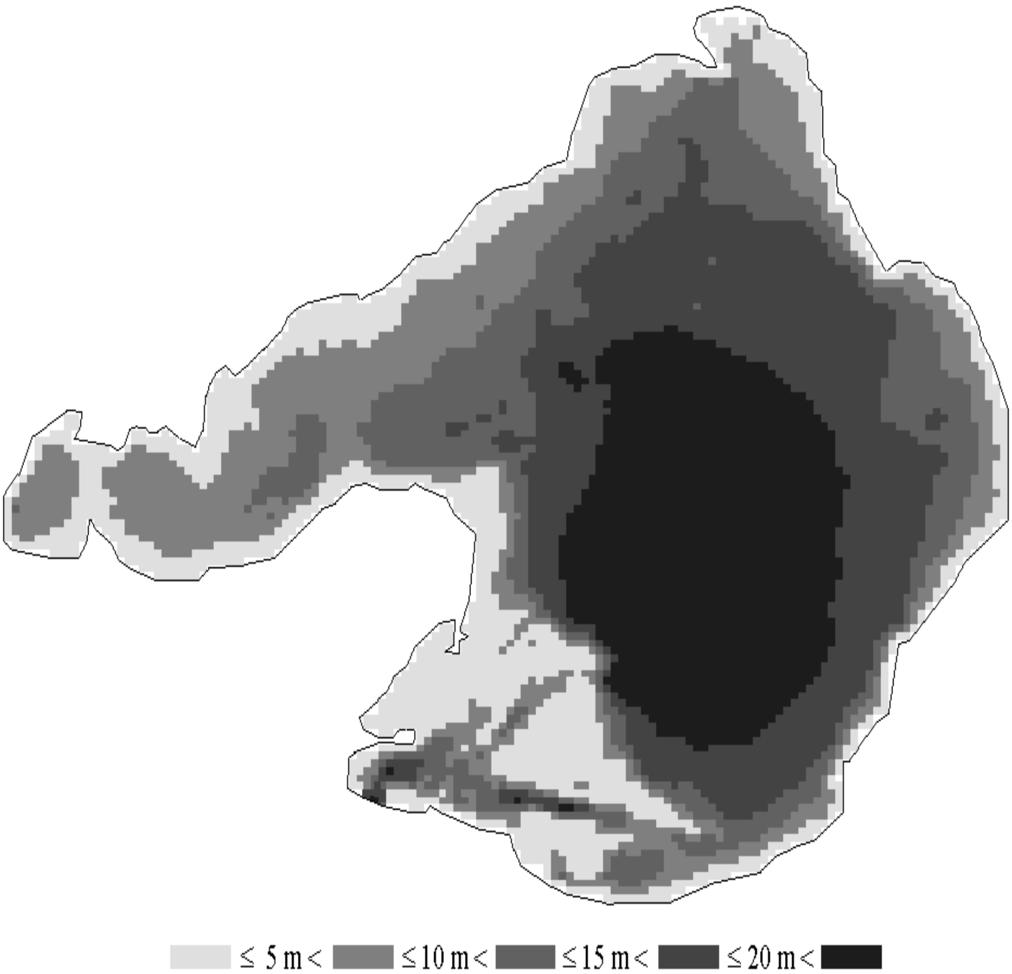


Figure 9. Interpolated yearly surface nutrient distributions (May 1993-April 1994)

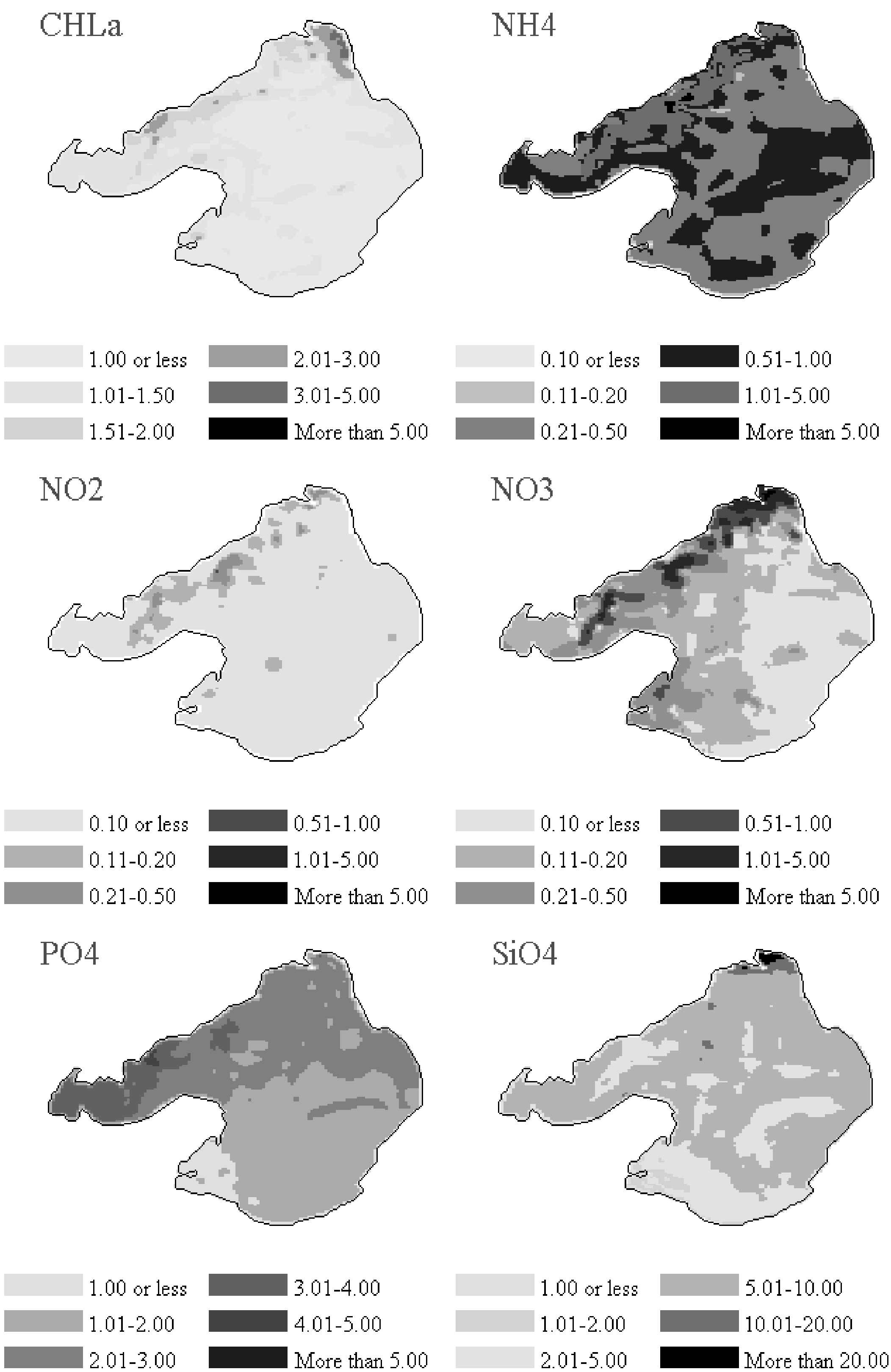


Figure 10. Interpolated yearly surface nutrient distributions (May 1994-March 1995)

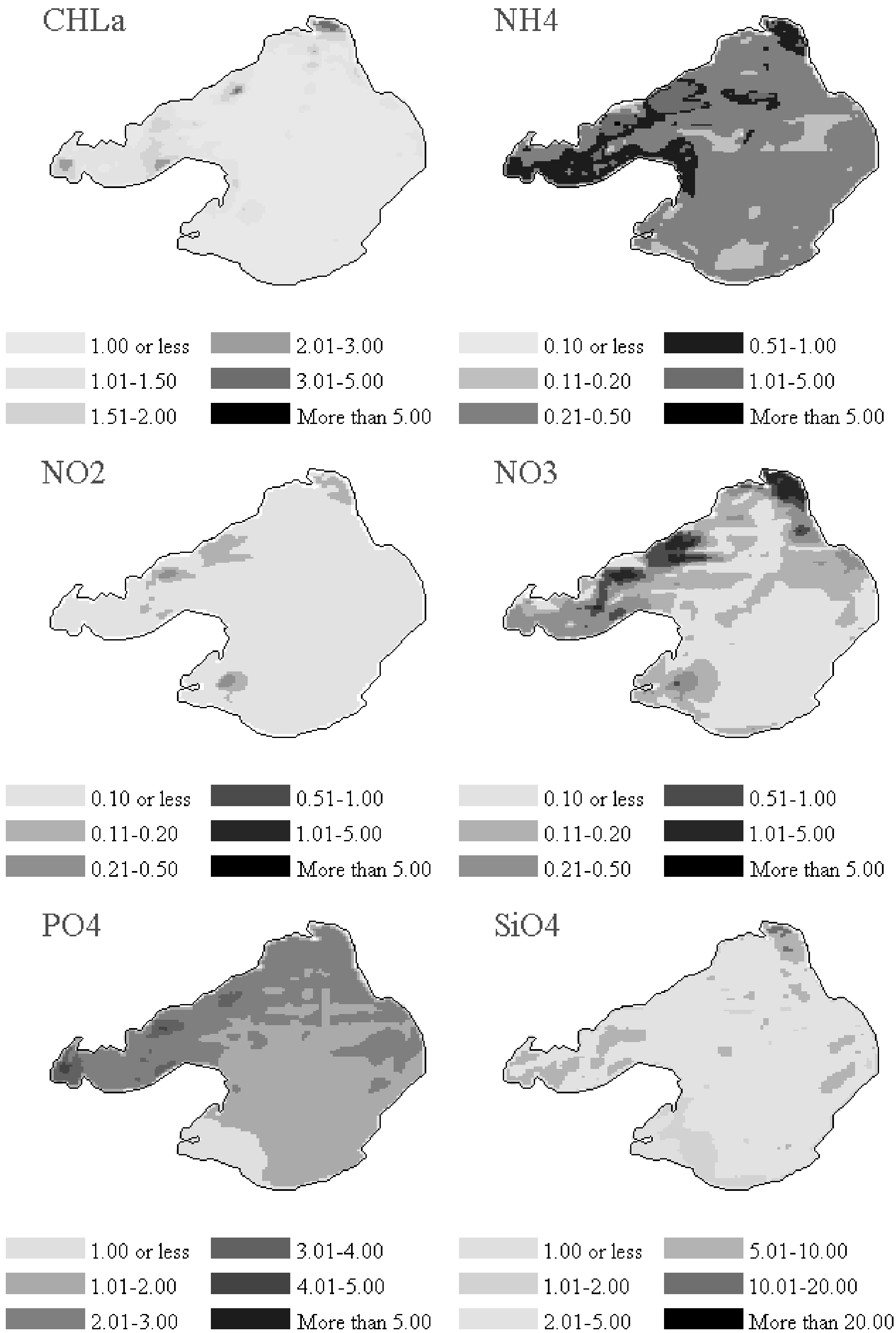


Figure 11. Interpolated S.E. of average surface nutrient distributions (May 1993-April 1994)

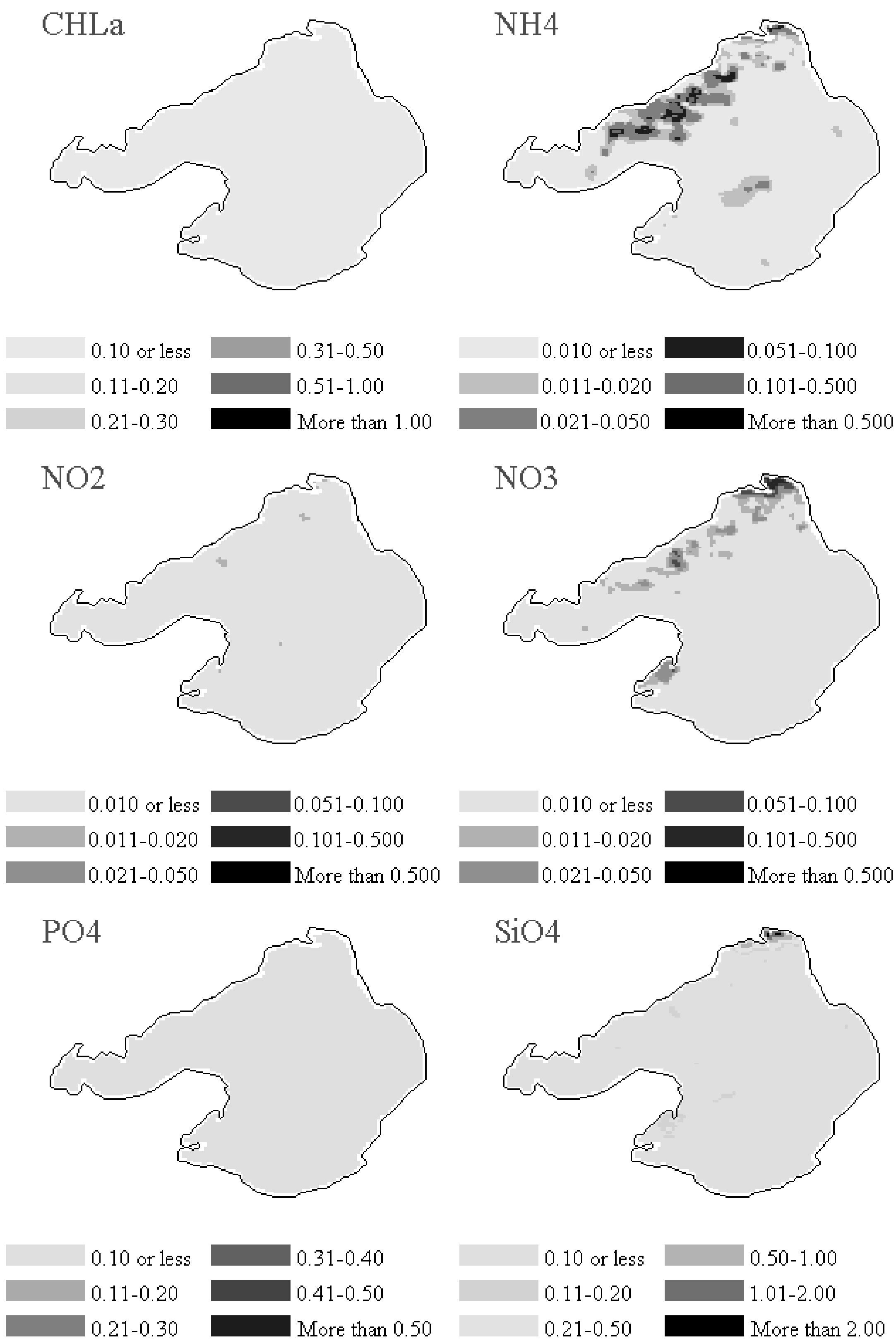


Figure 12. Interpolated S.E. of average surface nutrient distributions (May 1994-March 1995)

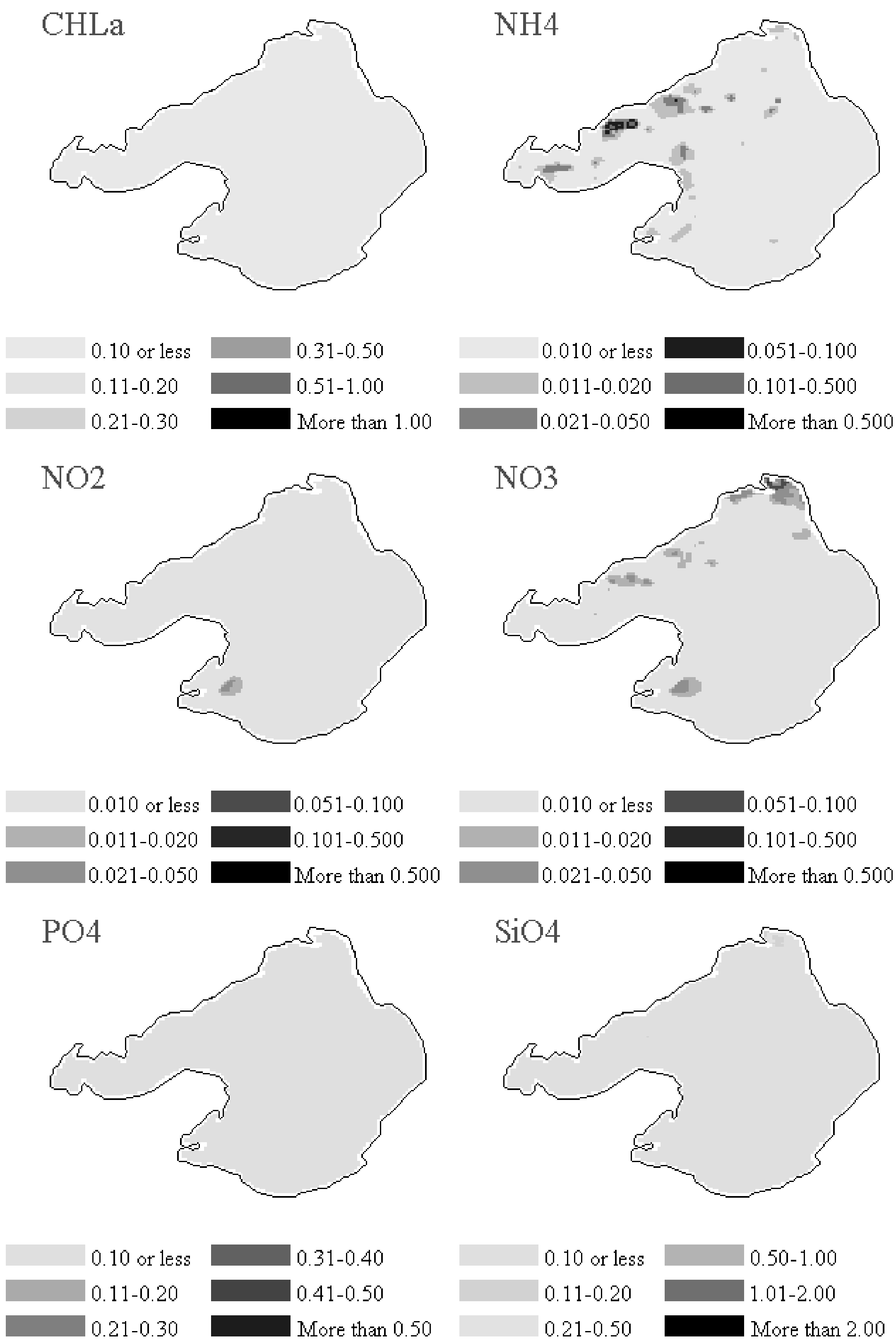


Figure13. Interpolated surface nutrient distributions (over all cruises)

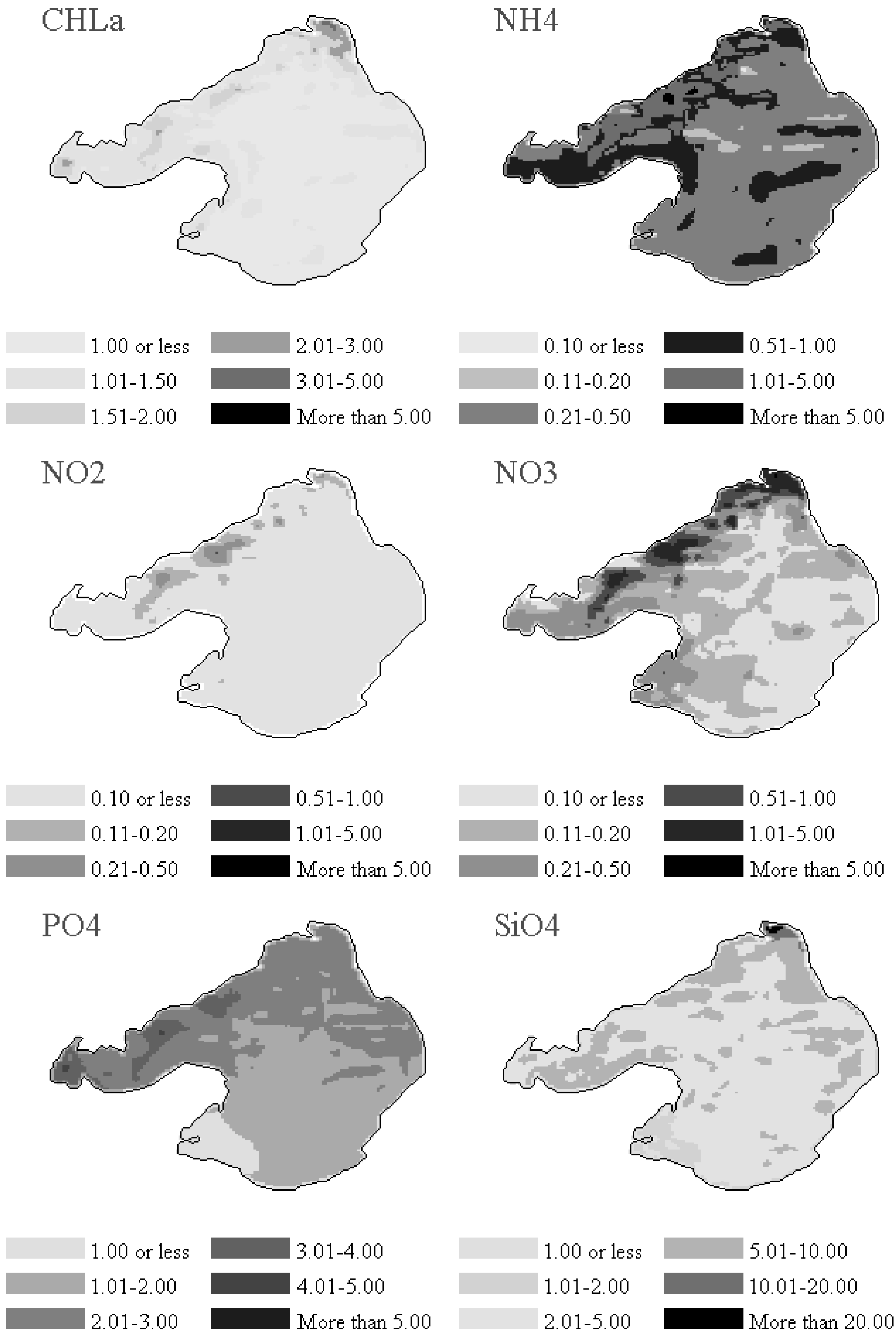


Figure 14. Interpolated S.E. of average surface nutrient distributions (over all cruises)

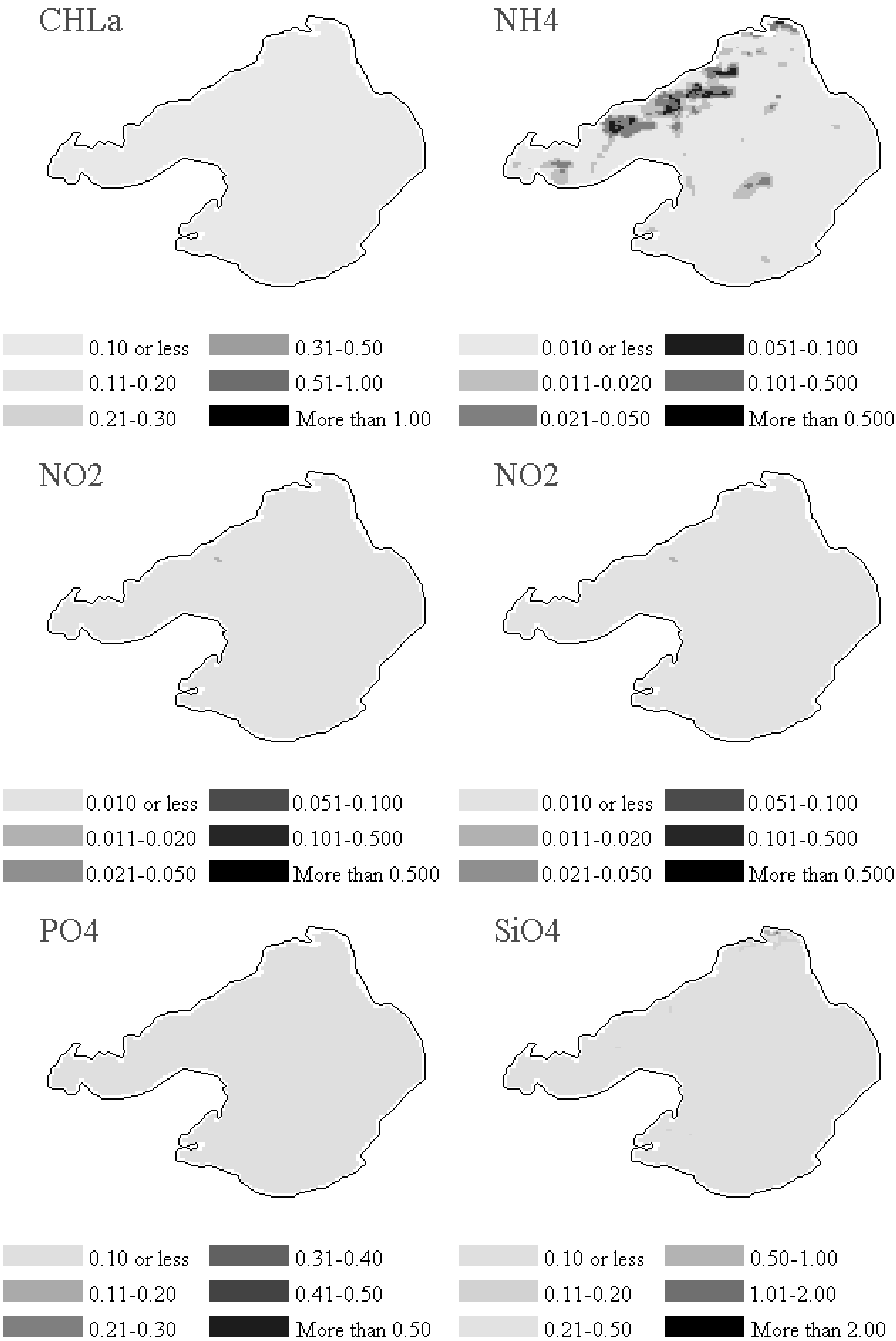


Figure 15. Interpolated surface Chlorophyll *a* distributions by cruises

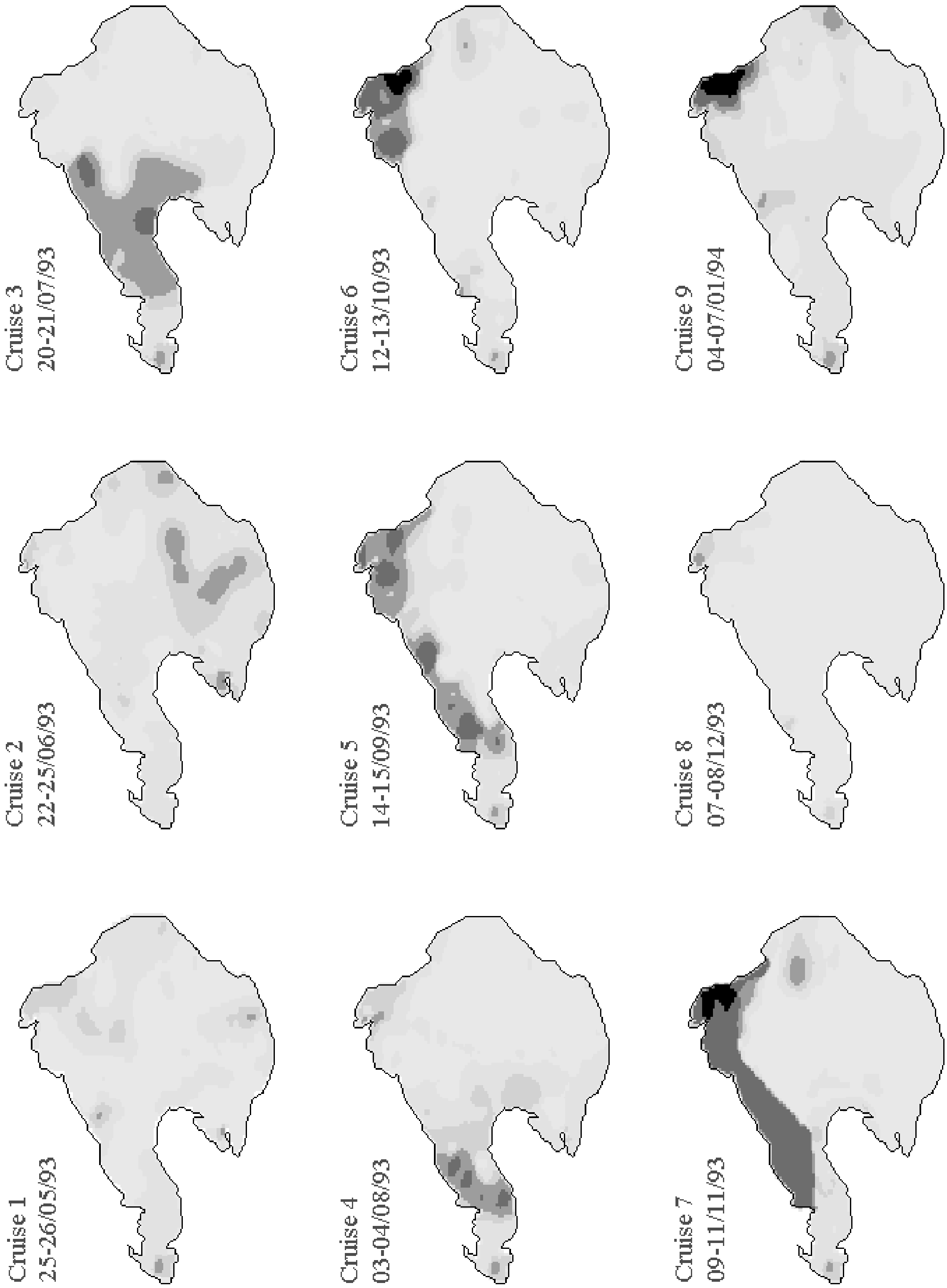


Figure 15.(cont.) Interpolated surface Chlorophyll *a* distributions by cruises

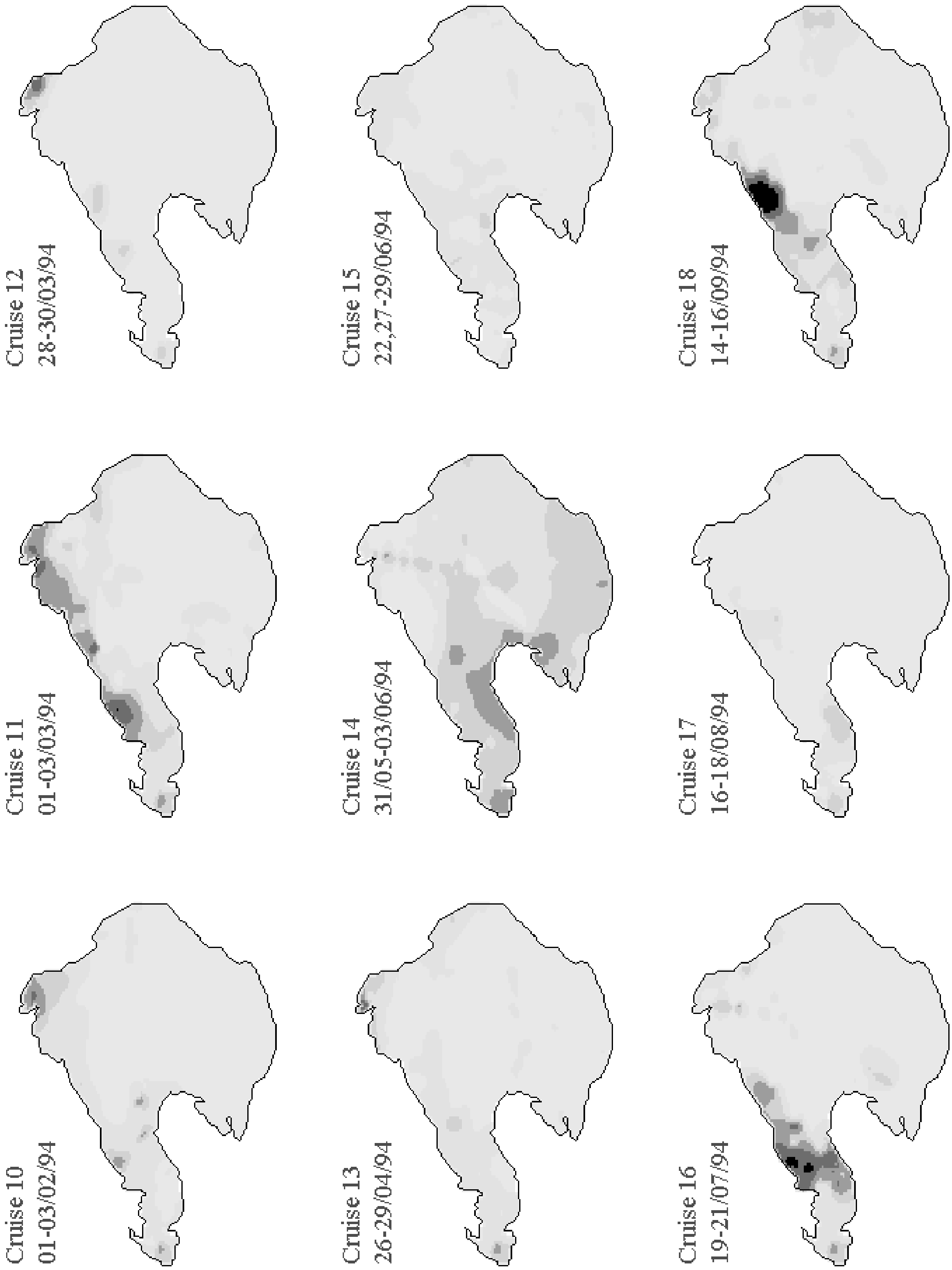


Figure 15.(cont.) Interpolated surface Chlorophyll _a_ distributions by cruises

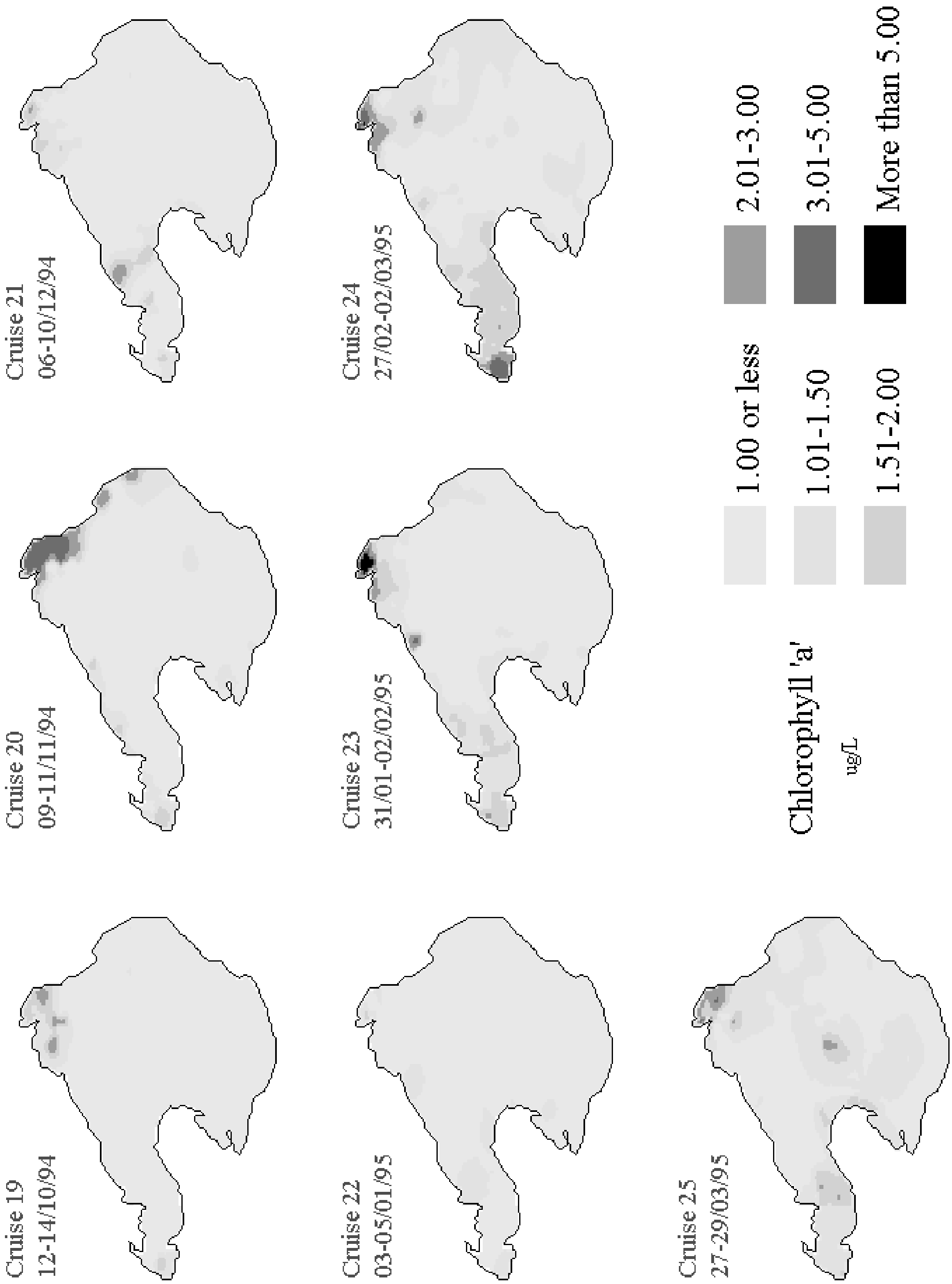


Figure 16. Interpolated surface Ammonium distributions by cruises

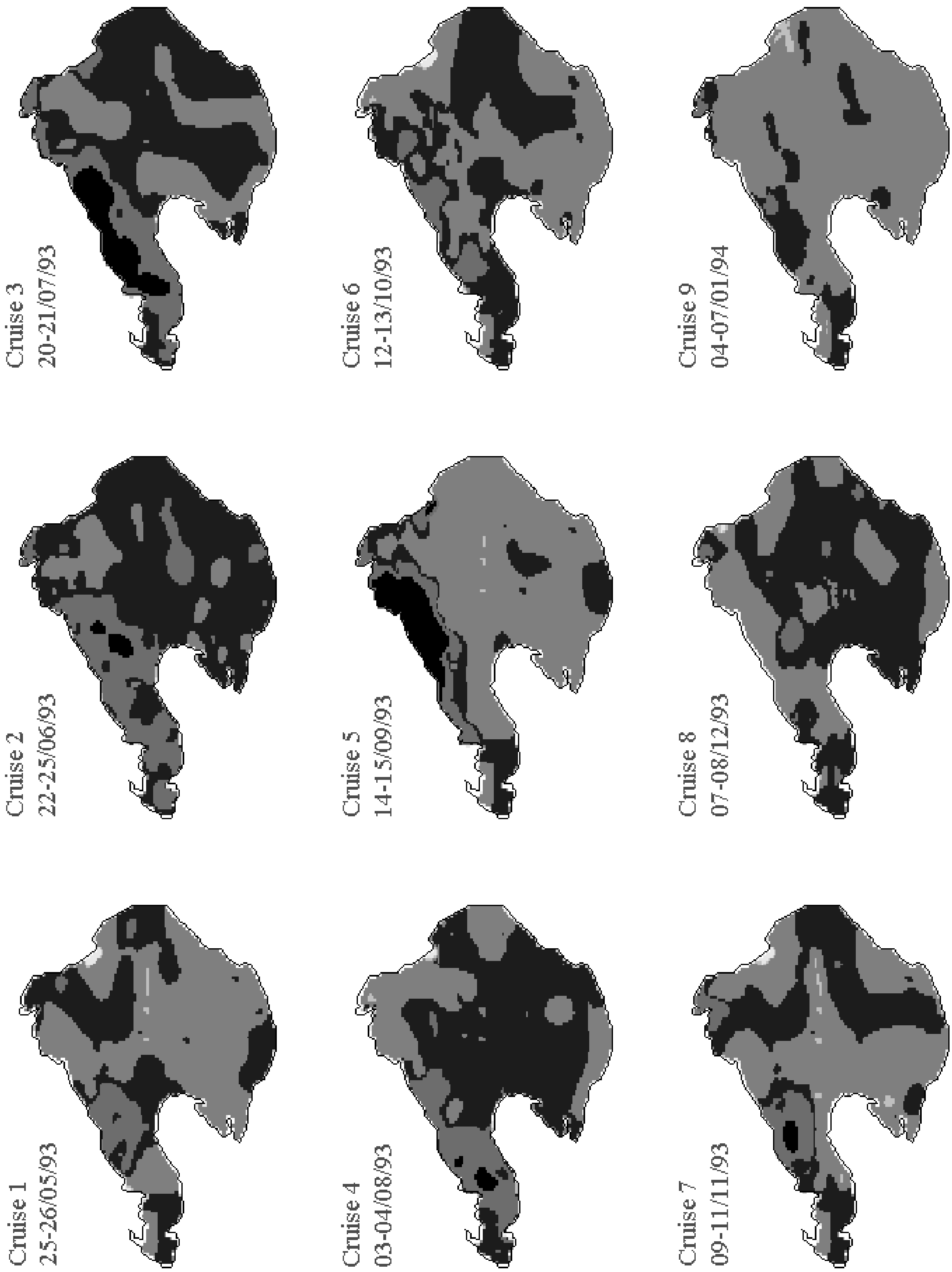


Figure 16.(cont.) Interpolated surface Ammonium distributions by cruises

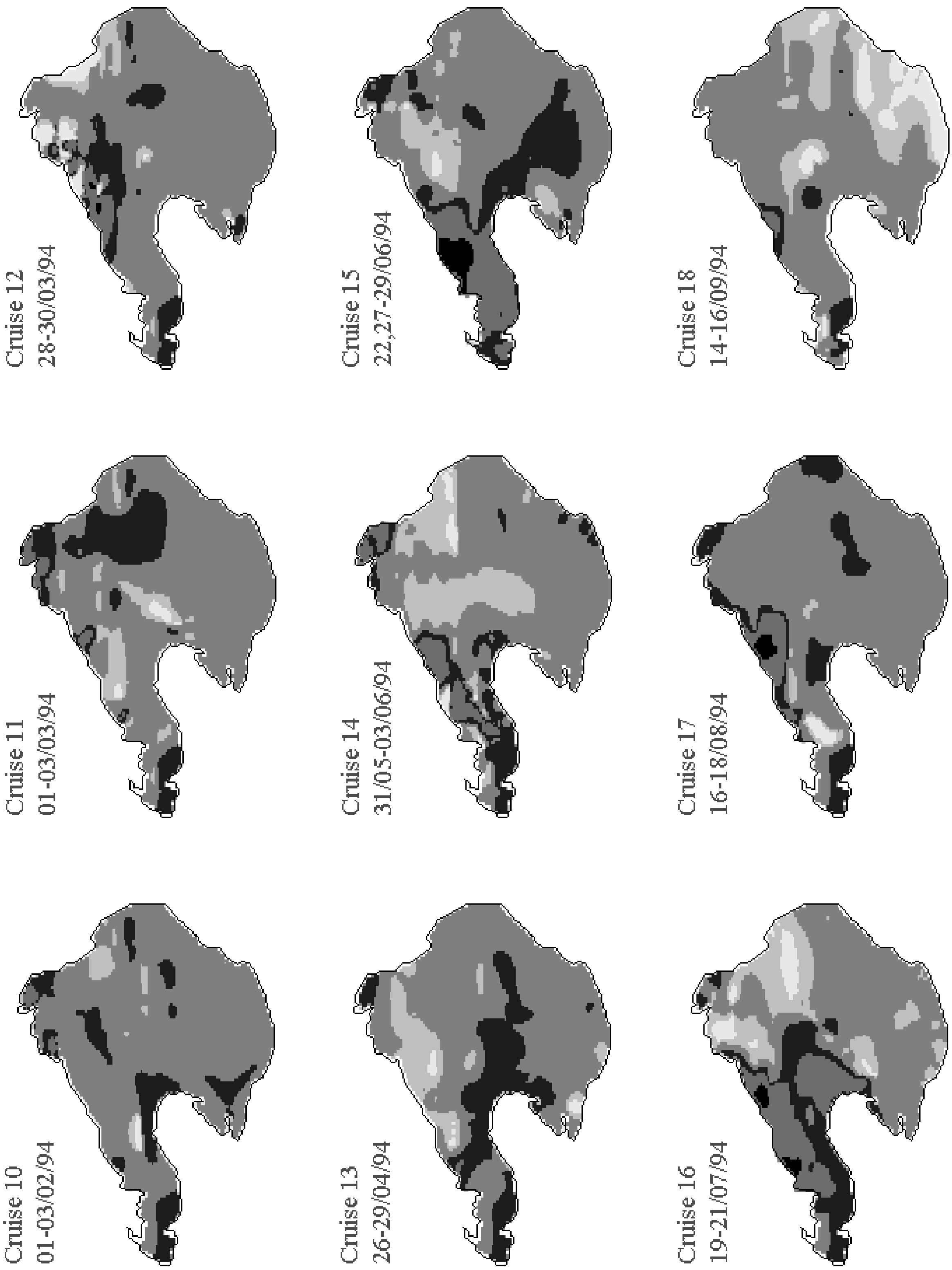


Figure 16.(cont.) Interpolated surface Ammonium distributions by cruises

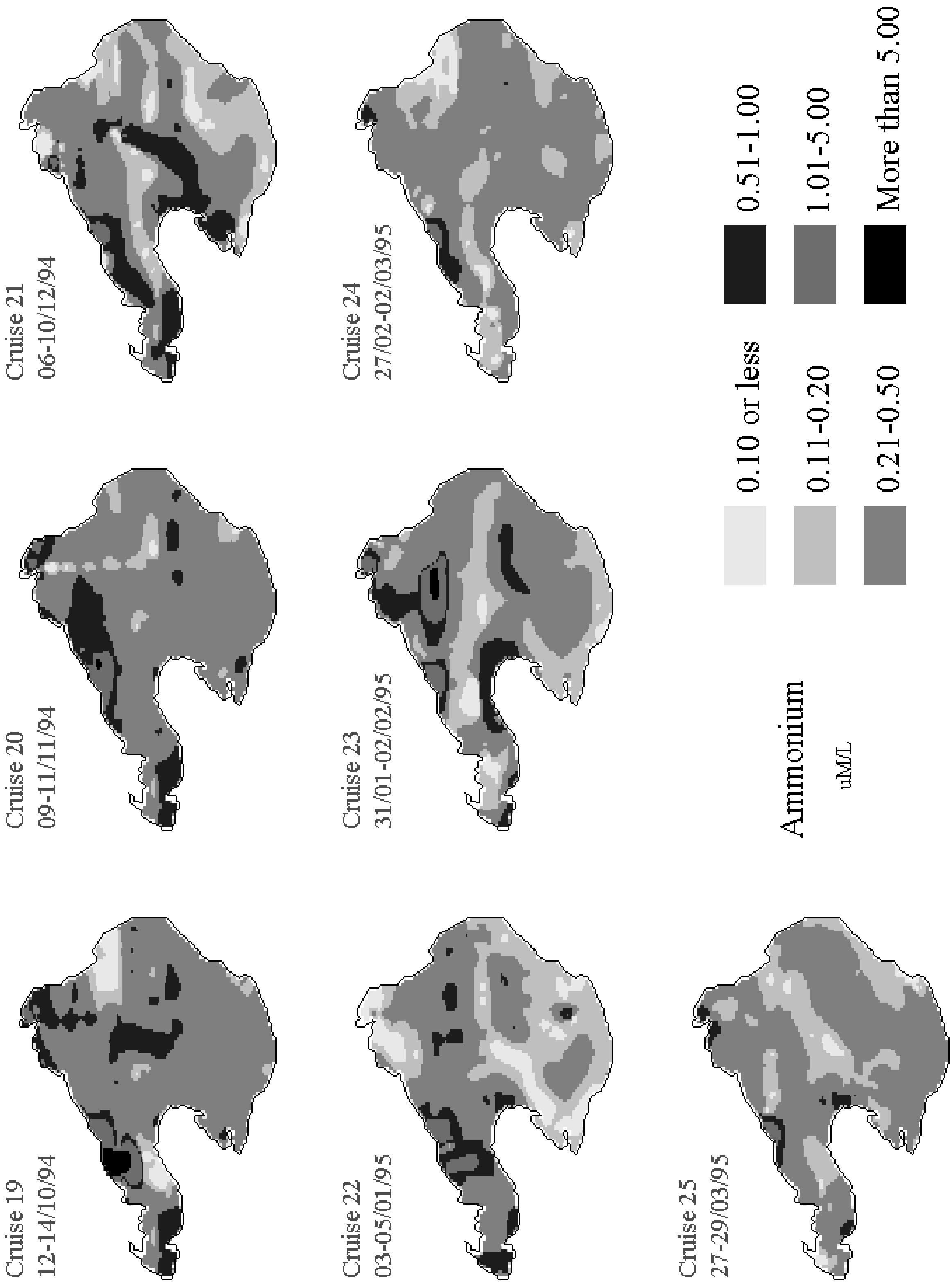


Figure 17. Interpolated surface Nitrite distributions by cruises

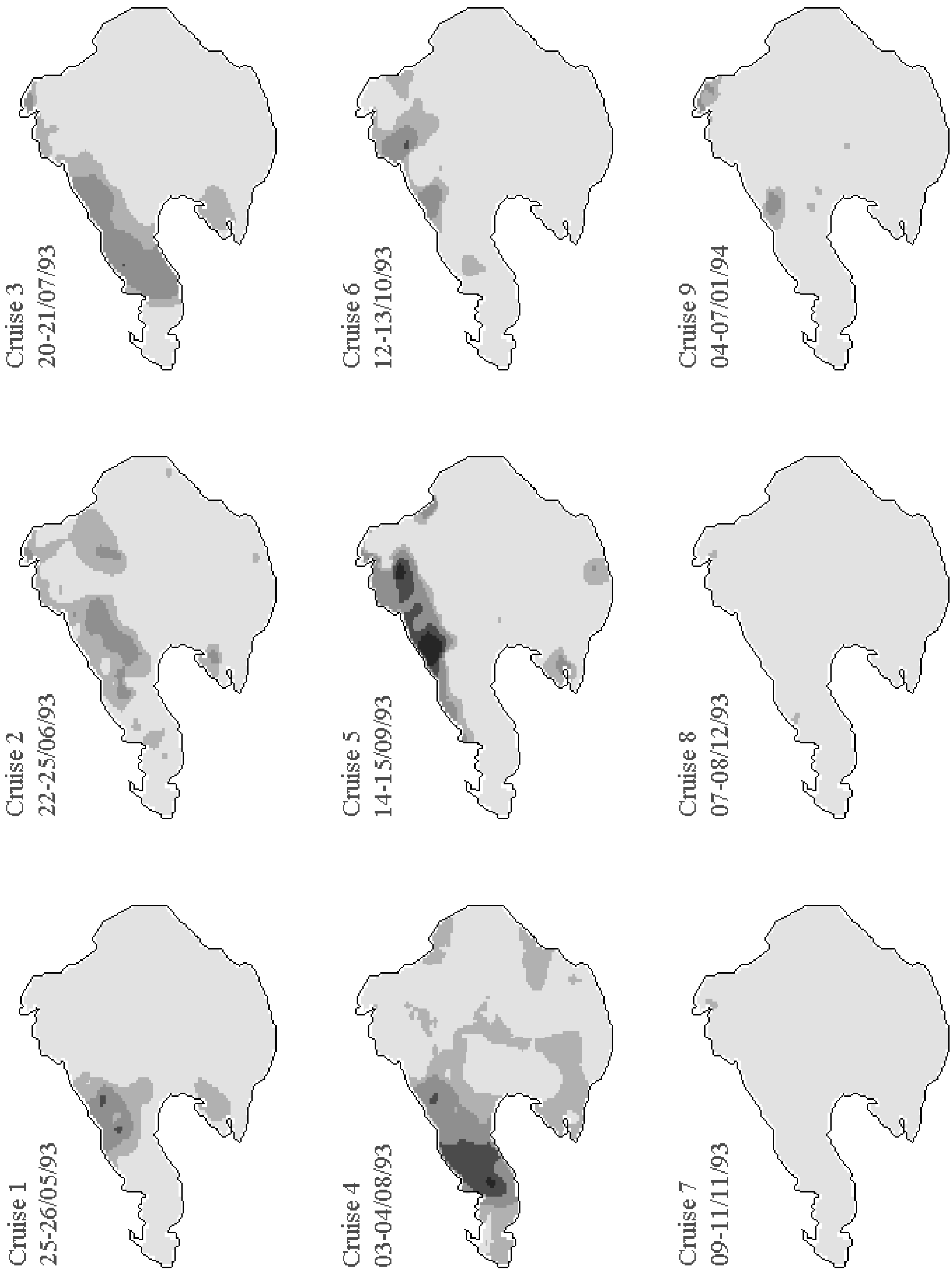


Figure 17.(cont.) Interpolated surface Nitrite distributions by cruises

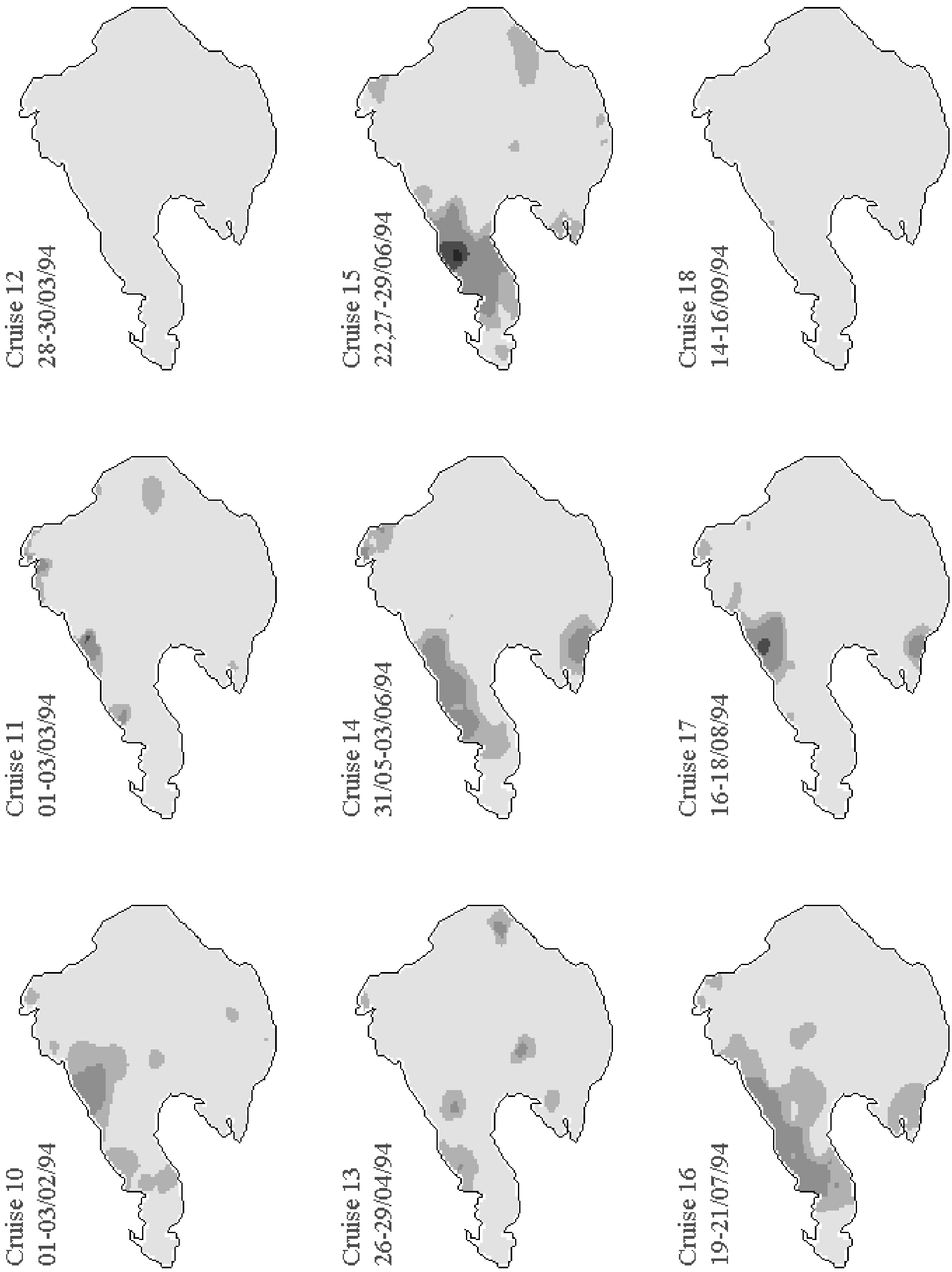


Figure 17.(cont.) Interpolated surface Nitrite distributions by cruises

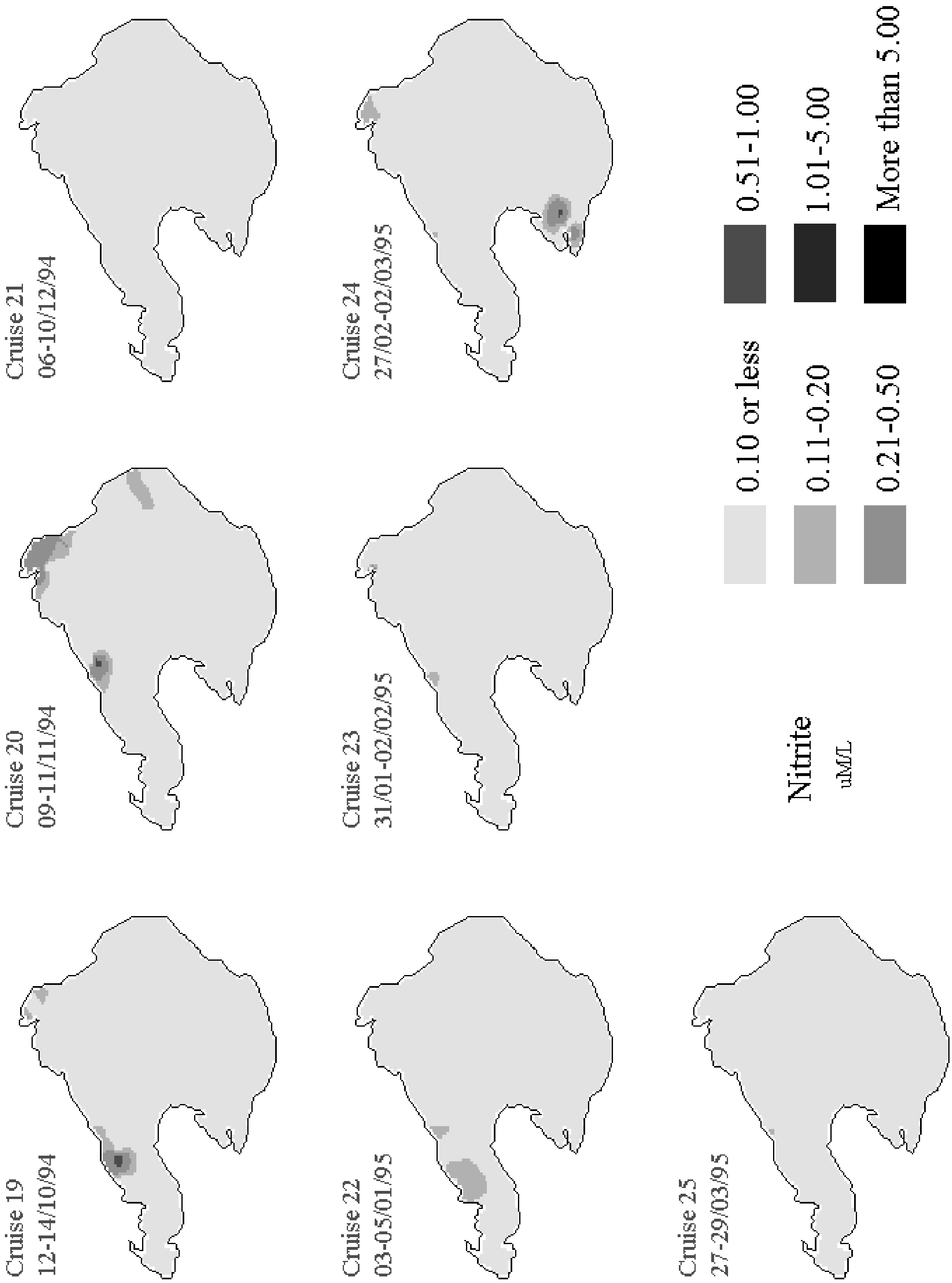


Figure 18. Interpolated surface Nitrate distributions by cruises

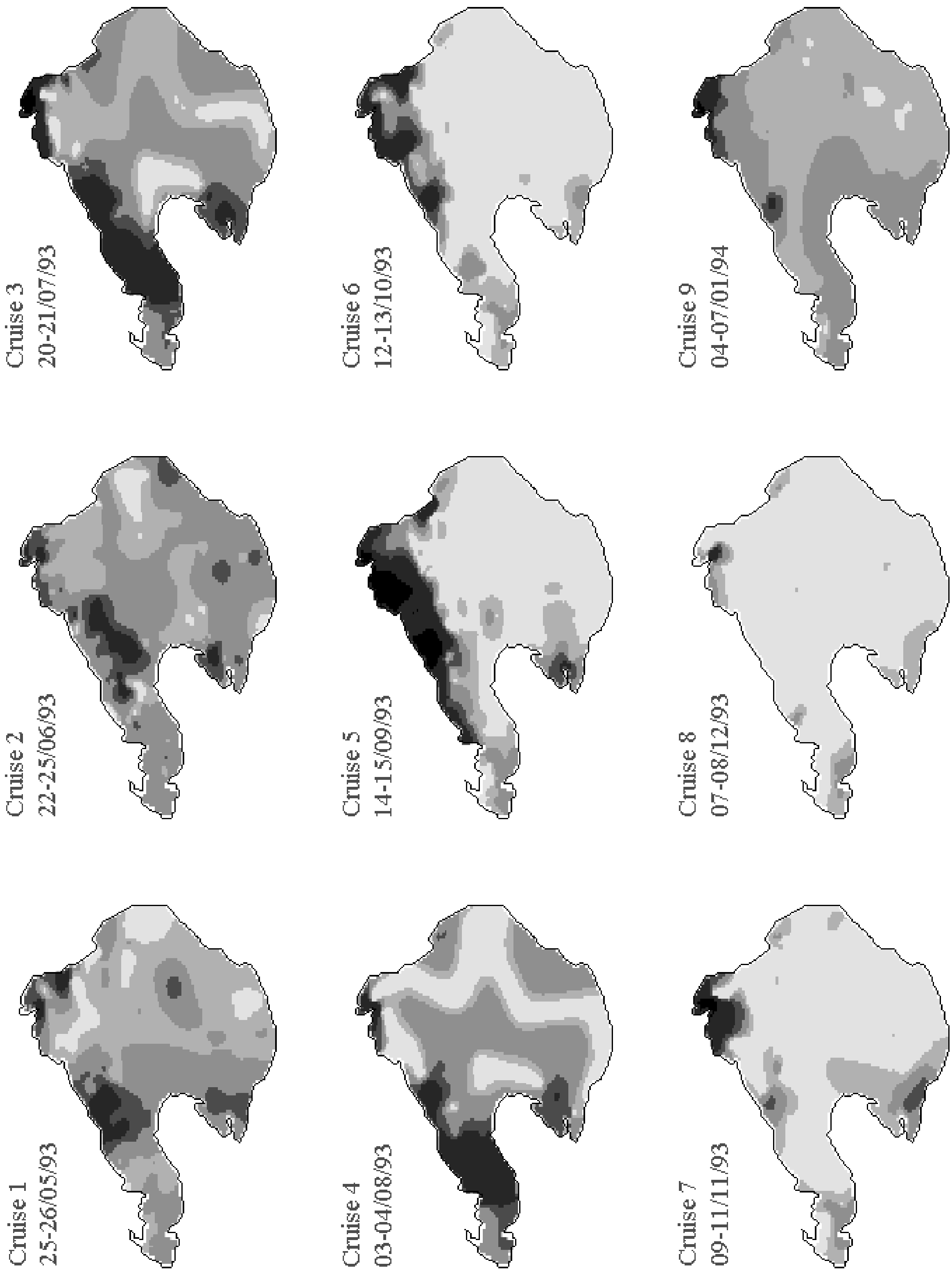


Figure 18.(cont.) Interpolated surface Nitrate distributions by cruises

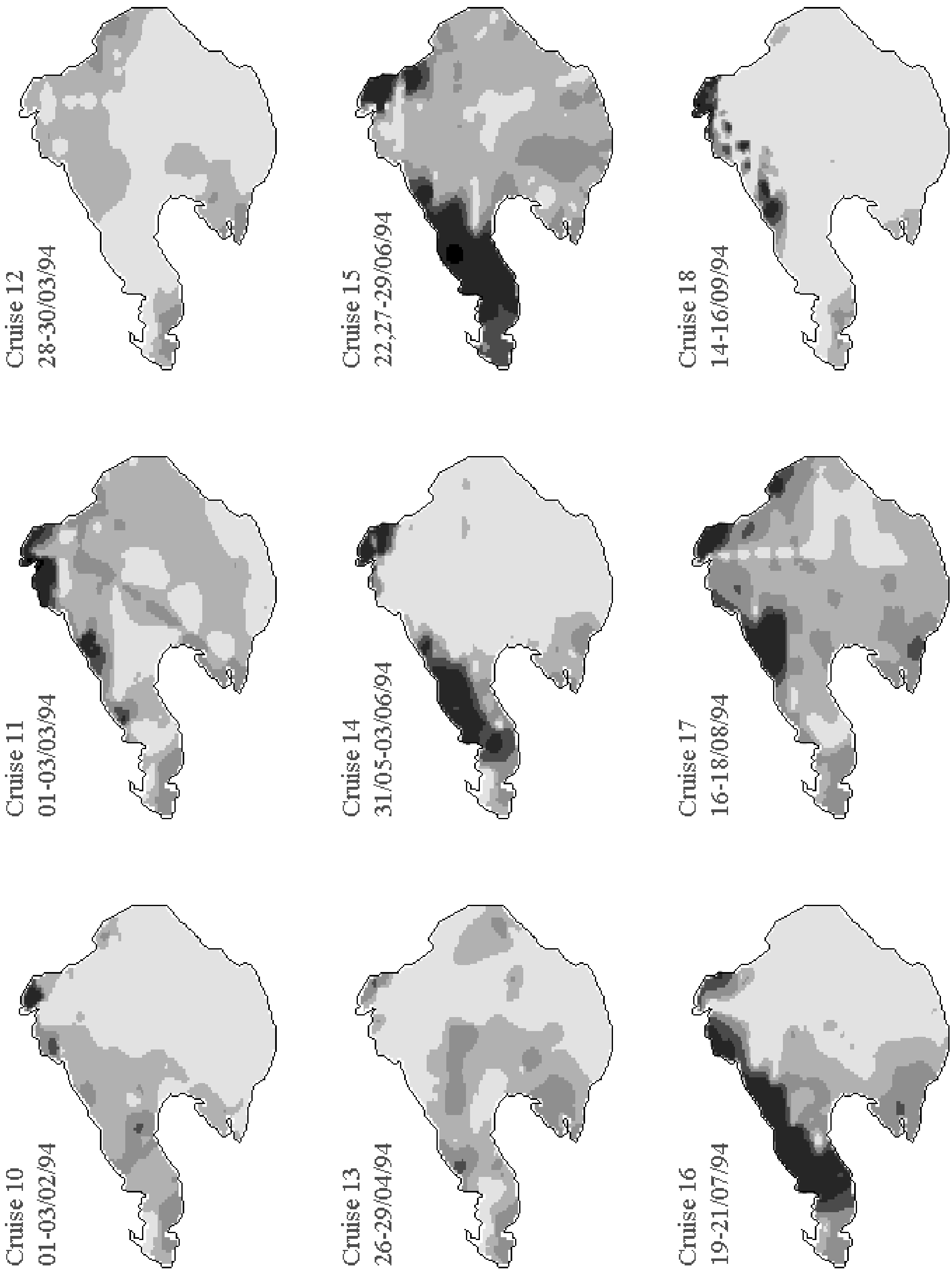


Figure 18.(cont.) Interpolated surface Nitrate distributions by cruises

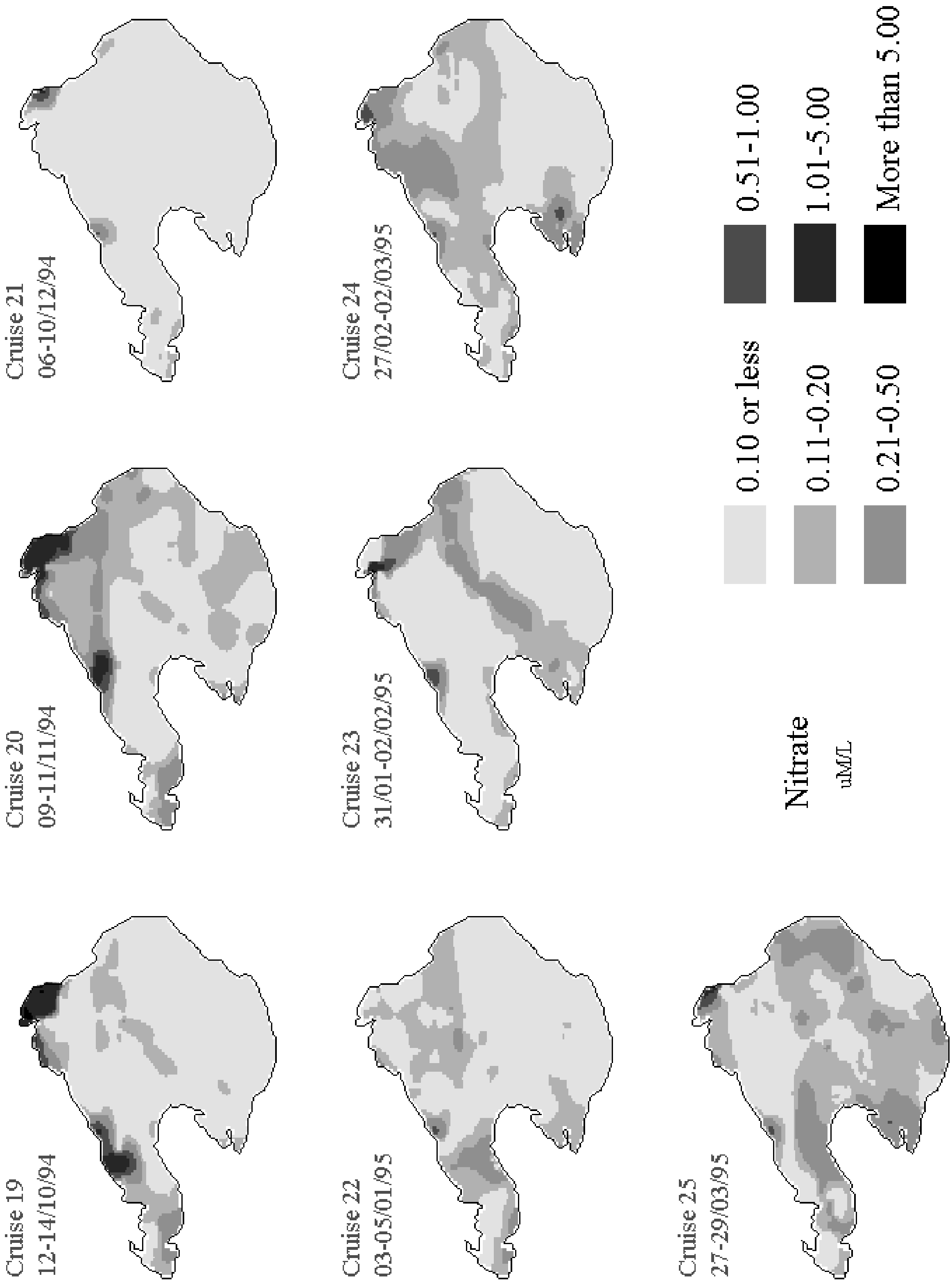


Figure 19. Interpolated surface Phosphate distributions by cruises

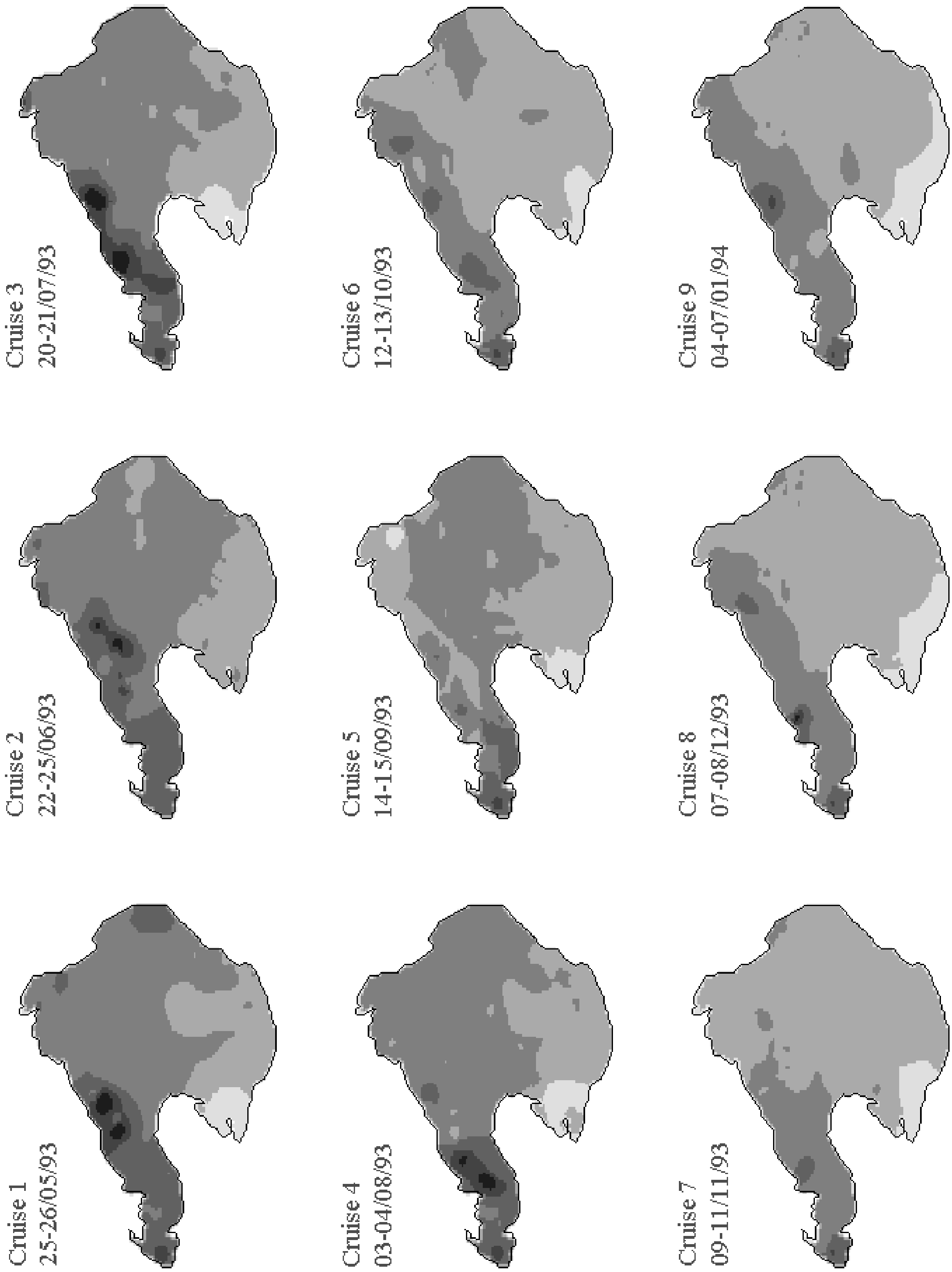


Figure 19.(cont.) Interpolated surface Phosphate distributions by cruises

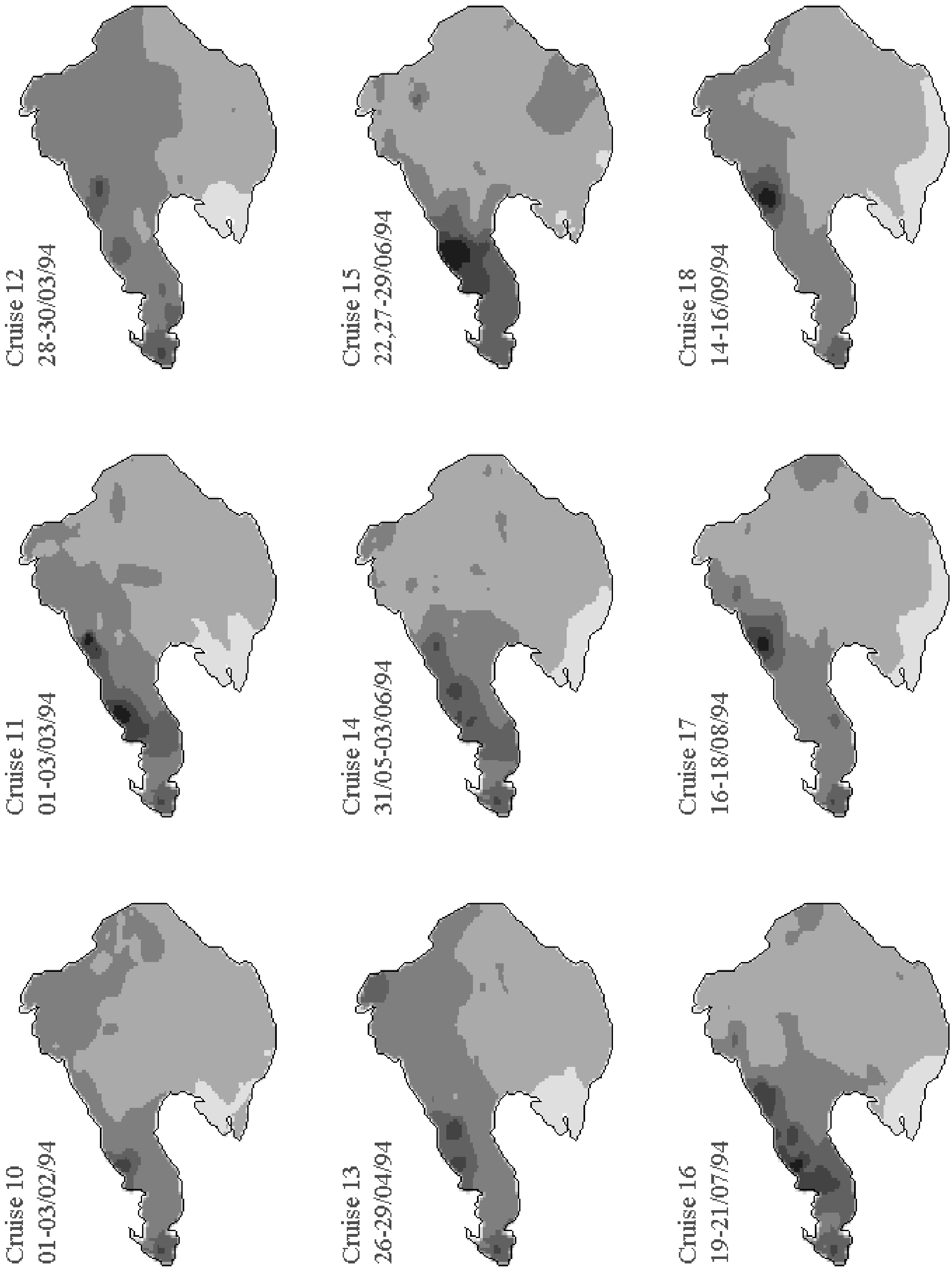


Figure 19.(cont.) Interpolated surface Phosphate distributions by cruises

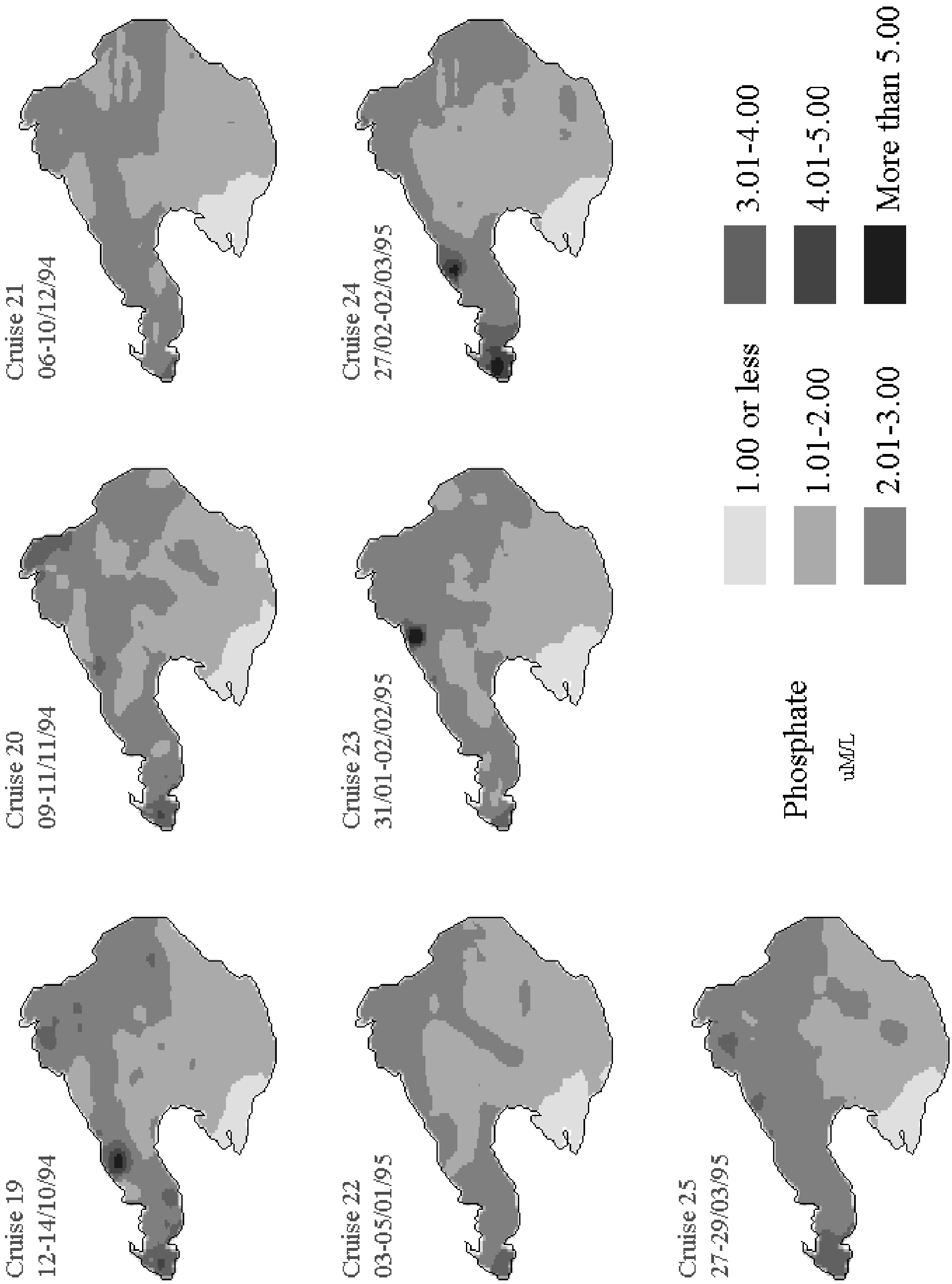


Figure 20. Interpolated surface Silicate distributions by cruises

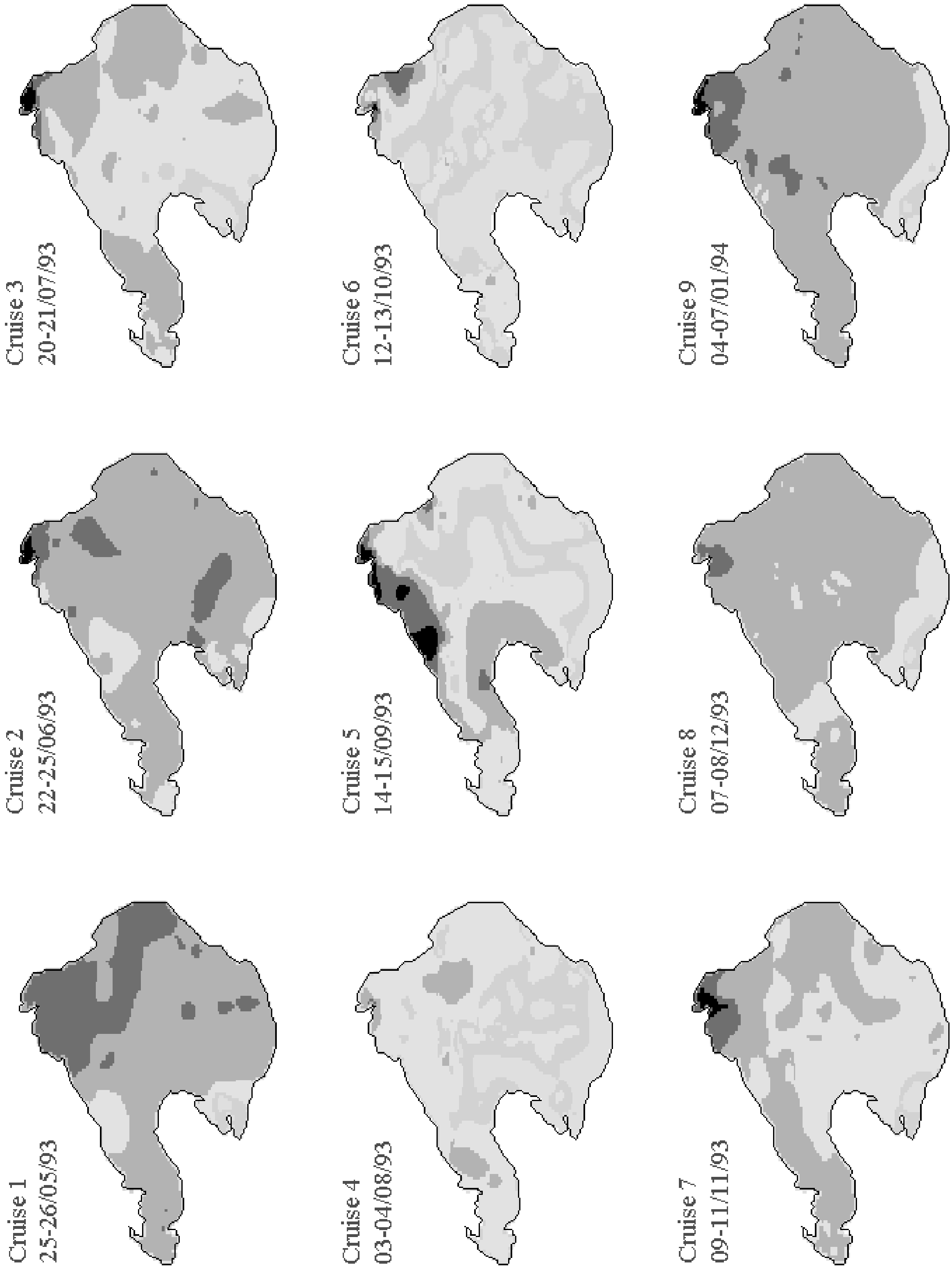


Figure 20.(cont.) Interpolated surface Silicate distributions by cruises

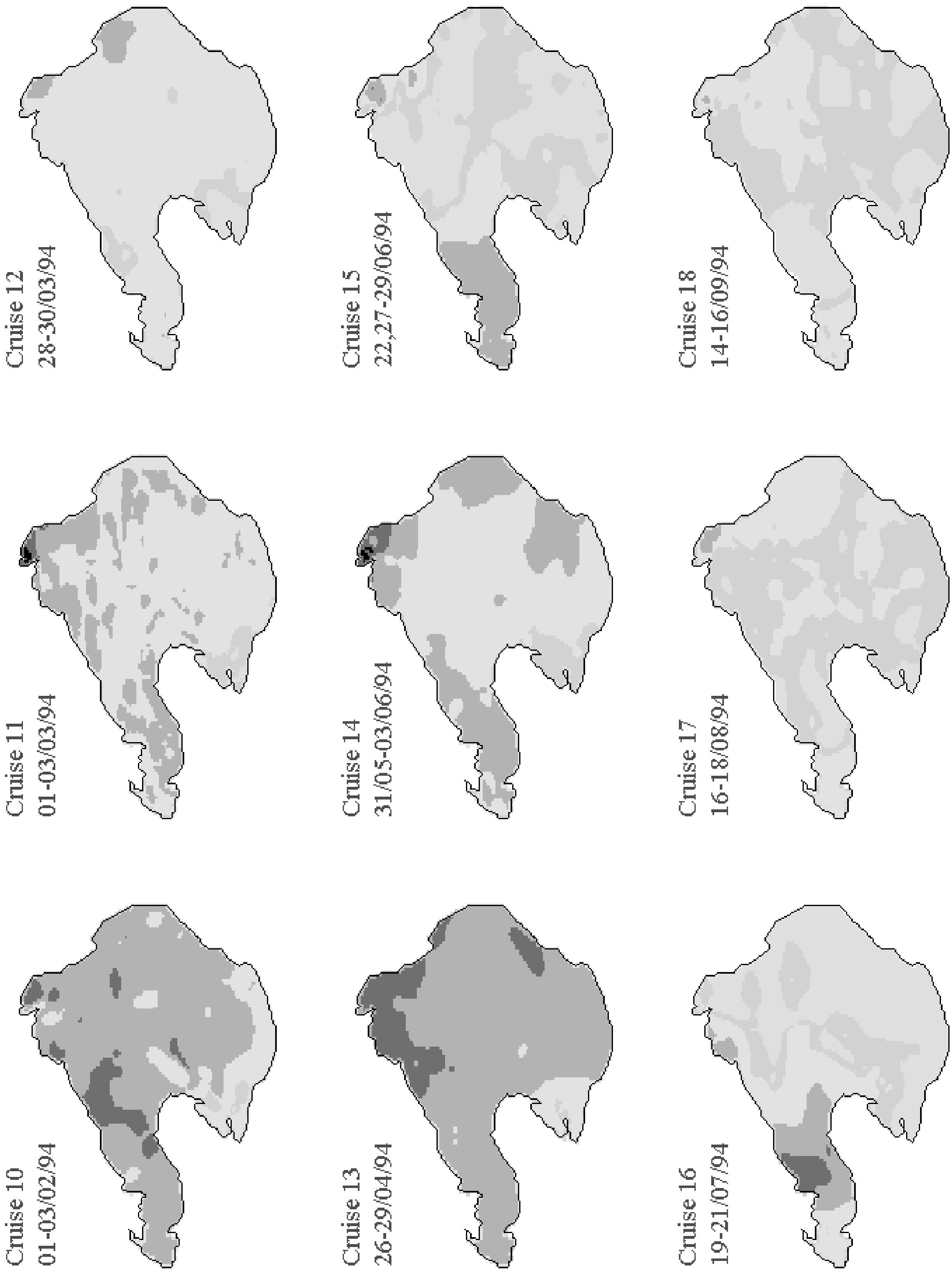


Figure 20.(cont.) Interpolated surface Silicate distributions by cruises

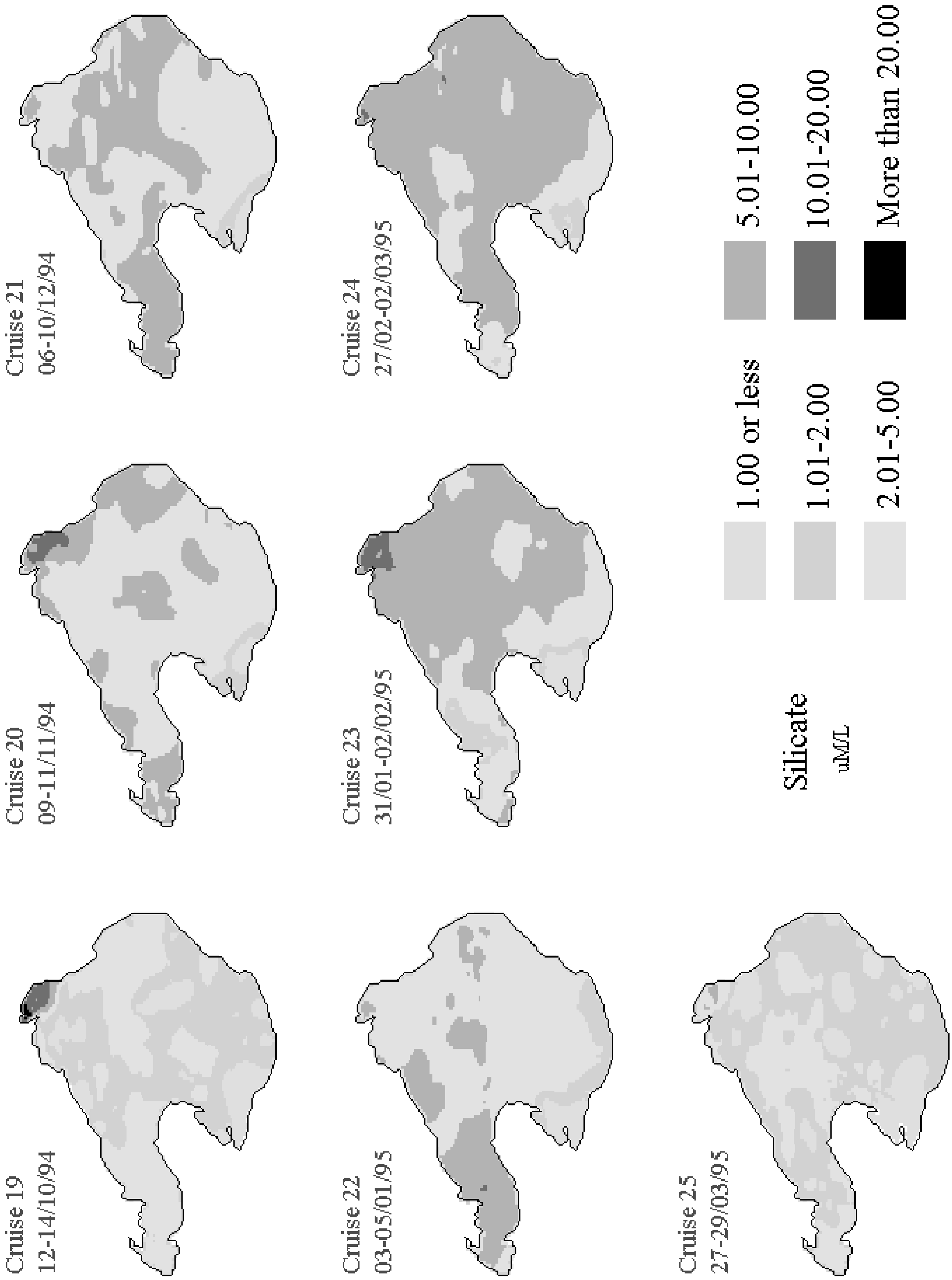


Figure 21. Interpolated S.E. of average Chlorophyll a distributions by cruises

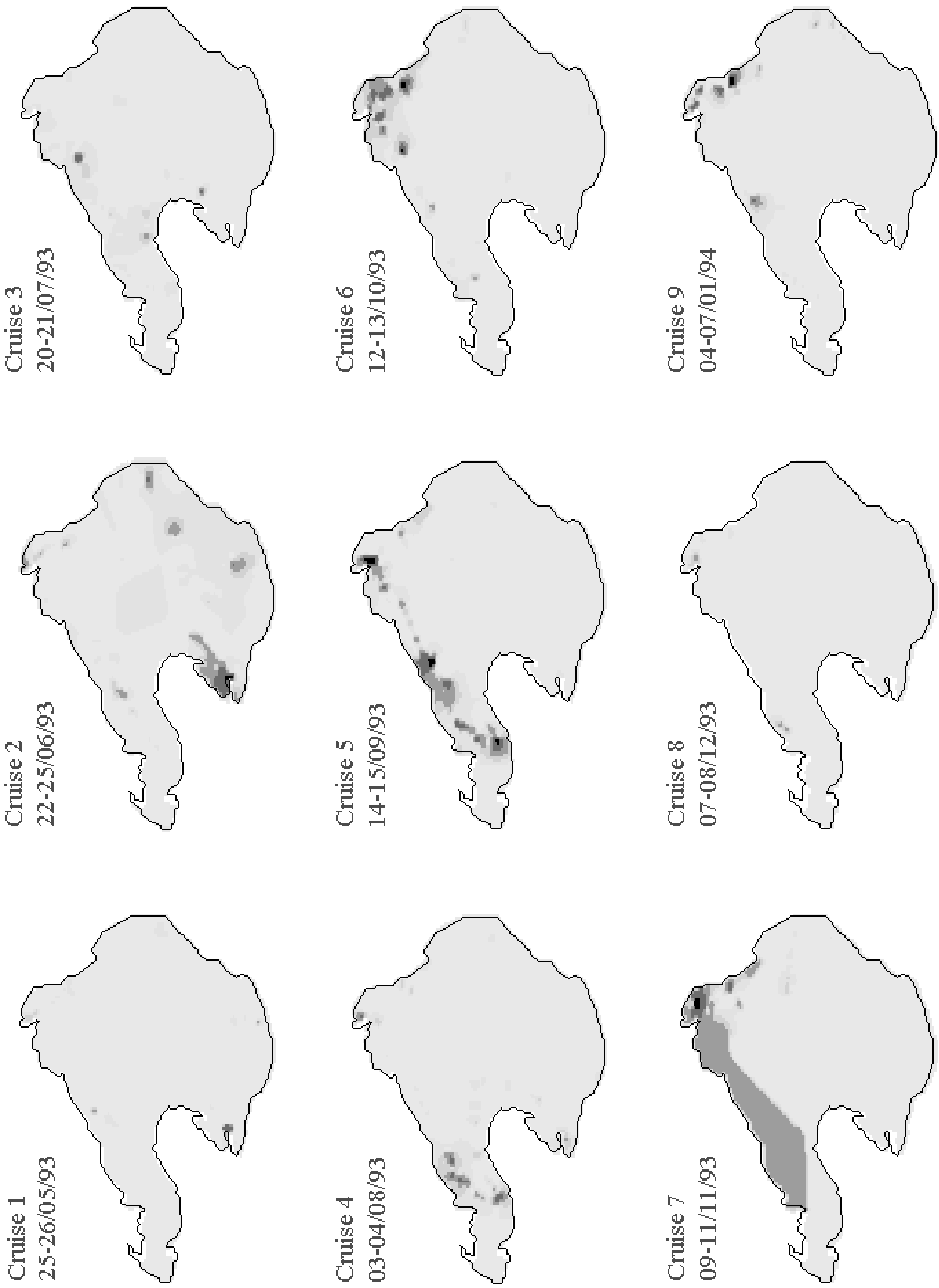


Figure 21.(cont.) Interpolated S.E. of average Chlorophyll _a_ distributions by cruises

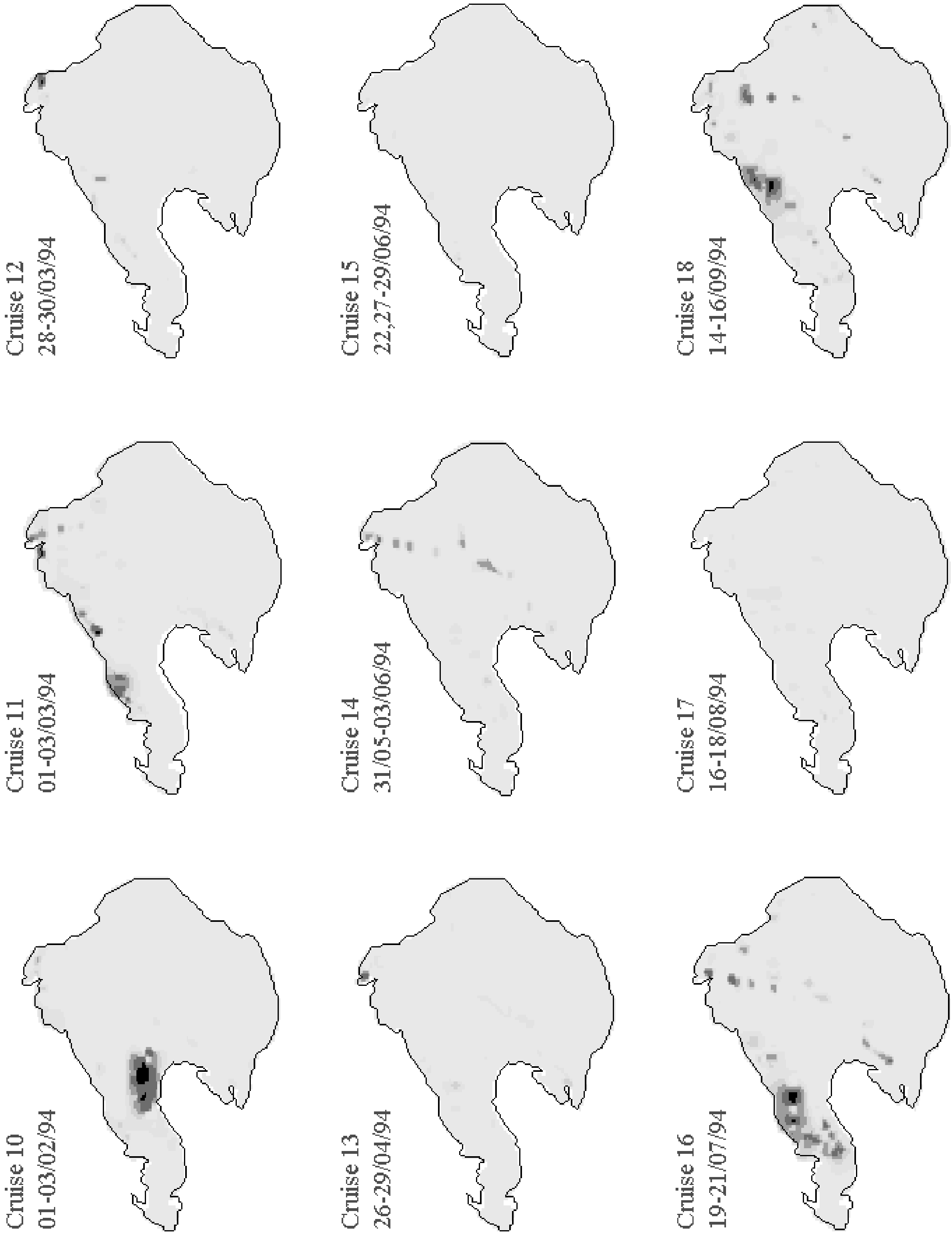


Figure 21.(cont.) Interpolated S.E. of average Chlorophyll _a_ distributions by cruises

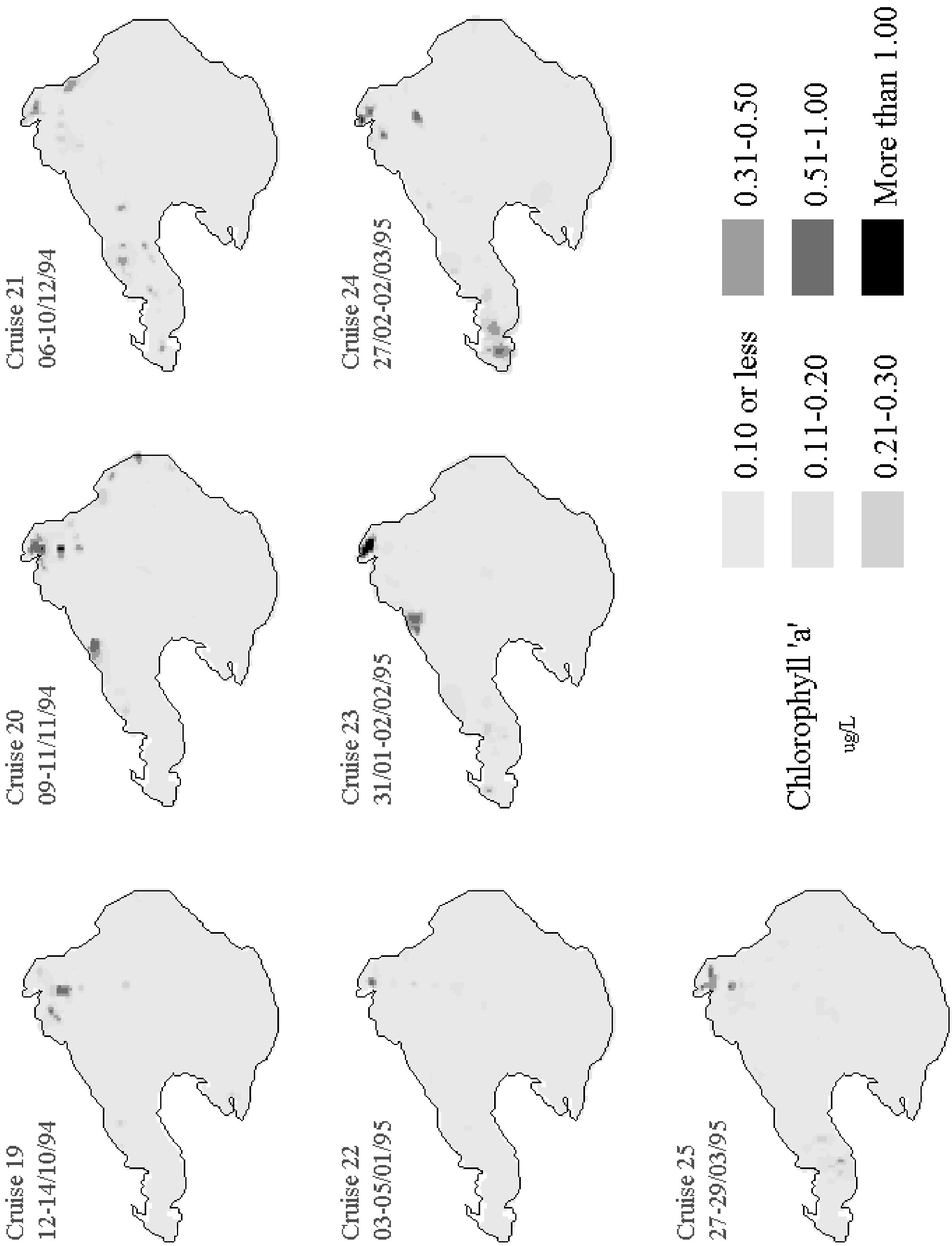


Figure 22. Interpolated S.E. of average Ammonium distributions by cruises

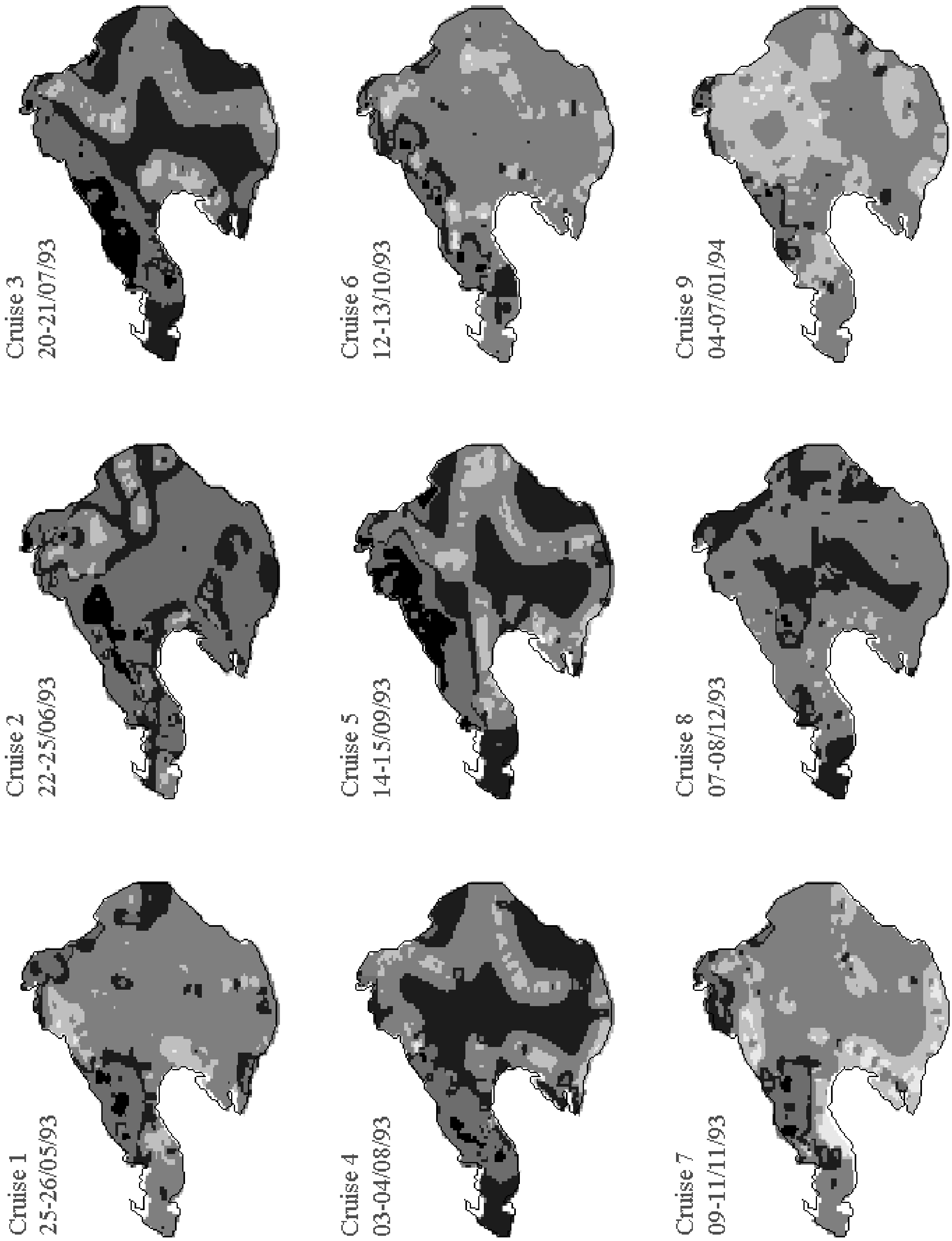


Figure 22.(cont.) Interpolated S.E. of average Ammonium distributions by cruises

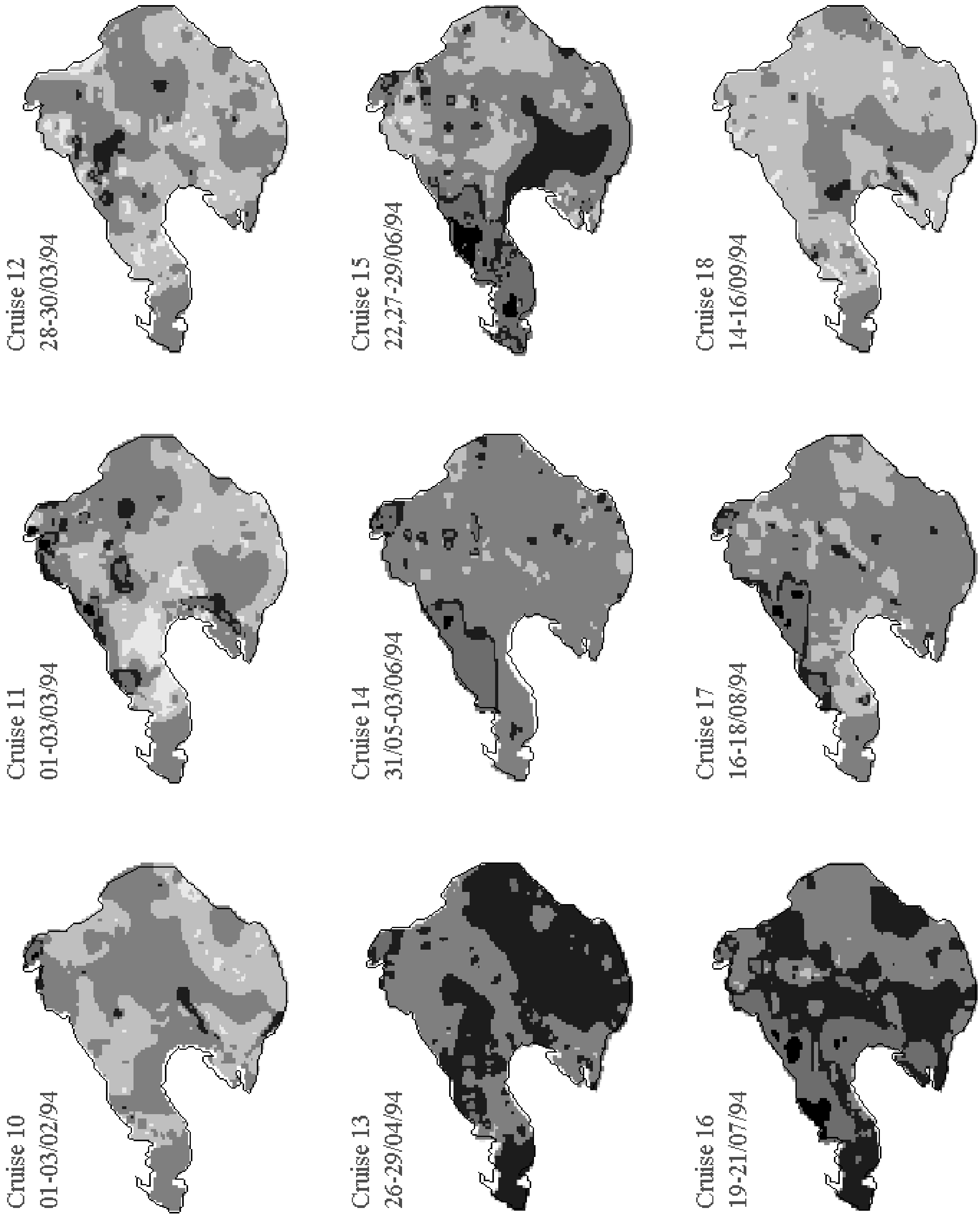


Figure 22.(cont.) Interpolated S.E. of average Ammonium distributions by cruises

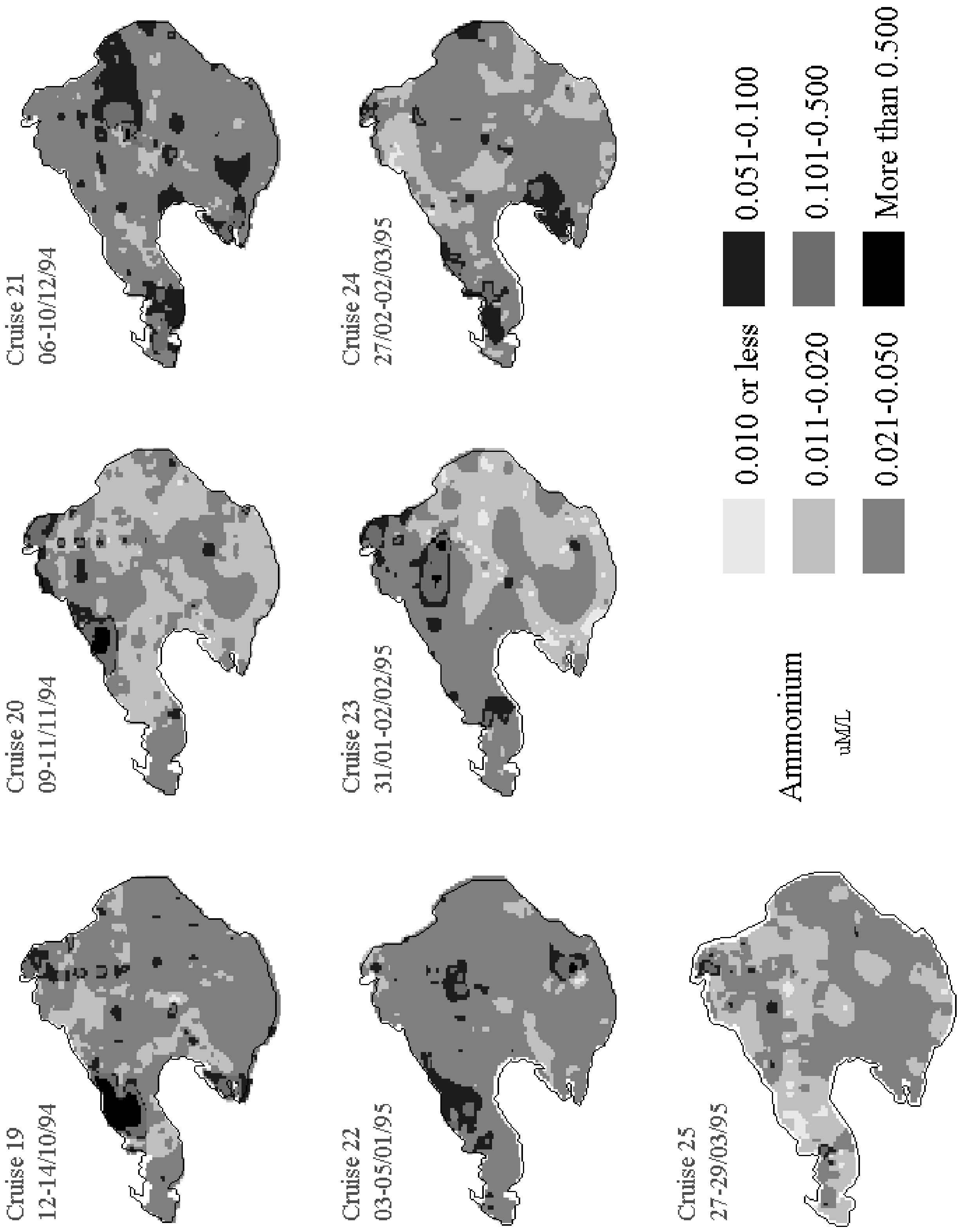


Figure 23. Interpolated S.E. of average Nitrite distributions by cruise

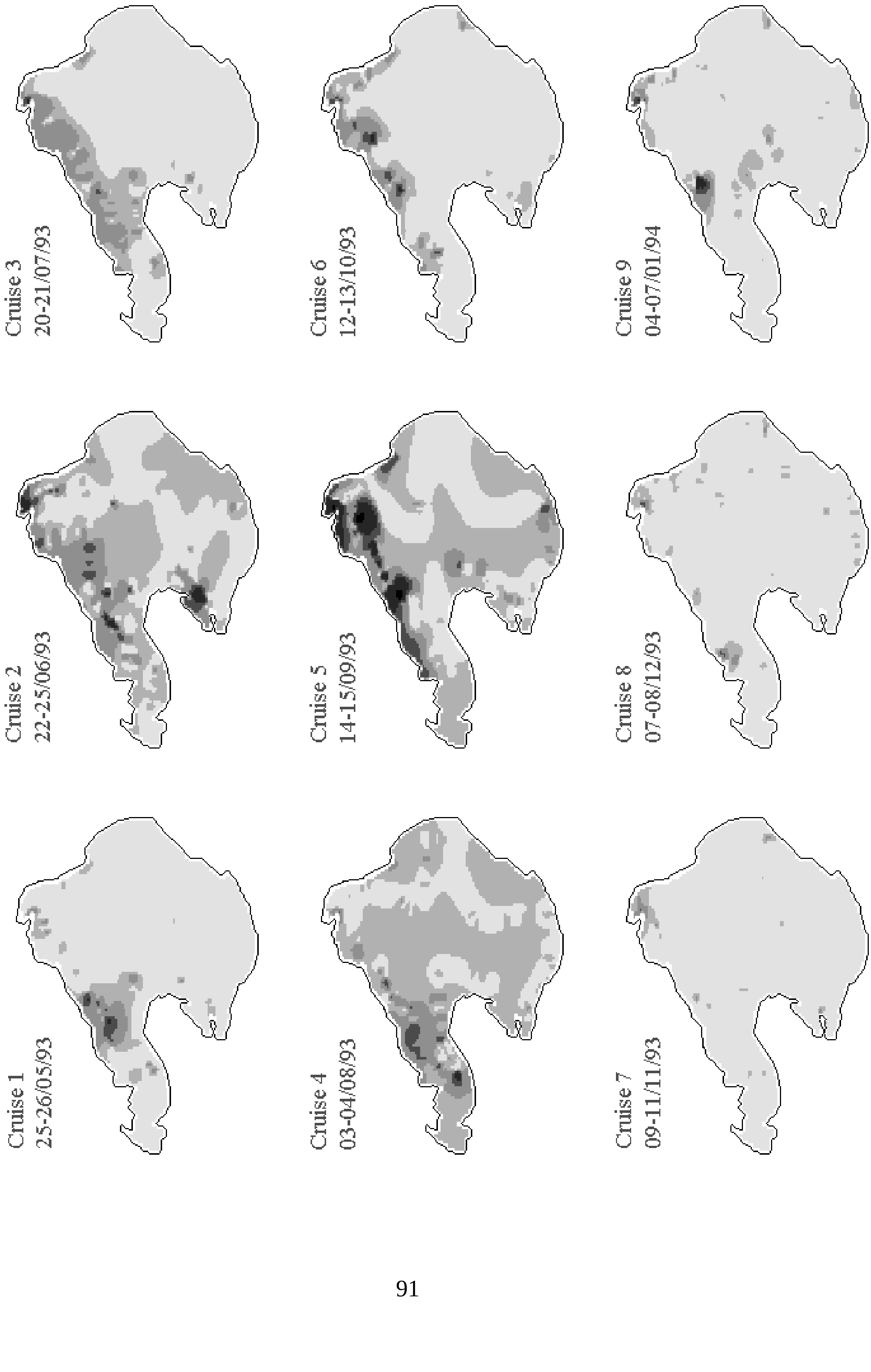


Figure 23.(cont.) Interpolated S.E. of average Nitrite distributions by cruise

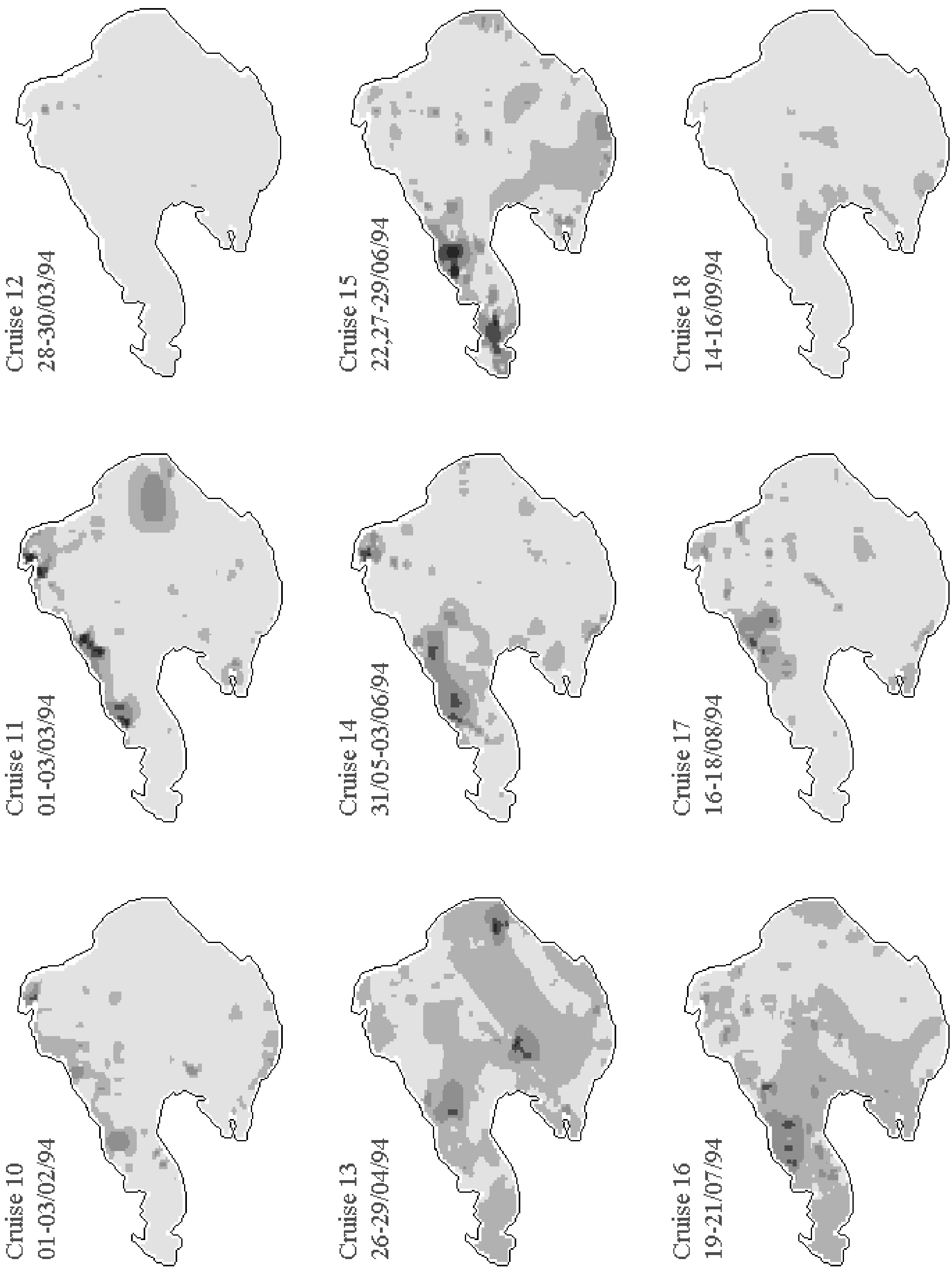


Figure 23.(cont.) Interpolated S.E. of average Nitrite distributions by cruise

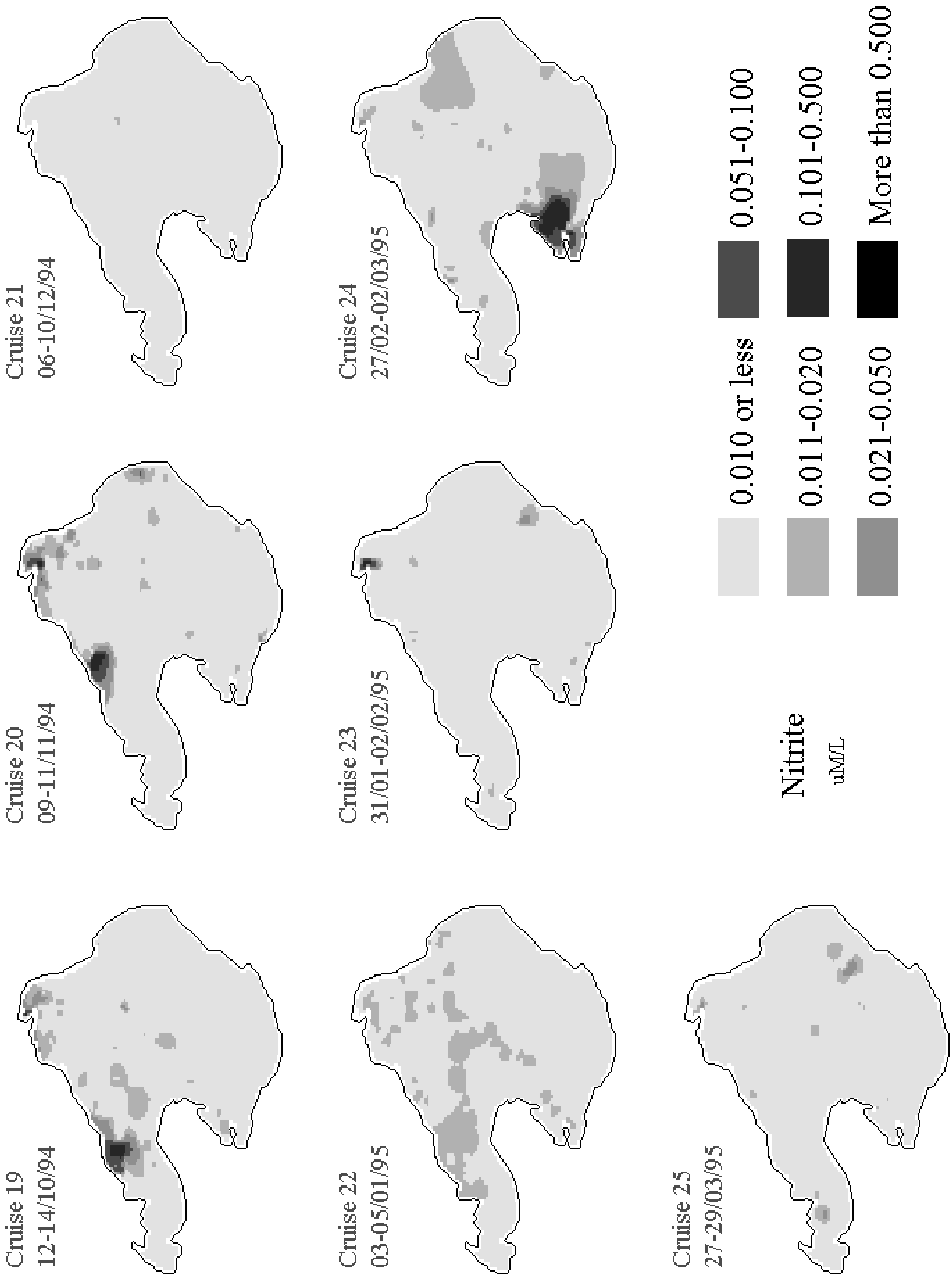


Figure 24. Interpolated S.E. of average Nitrate distributions by cruise

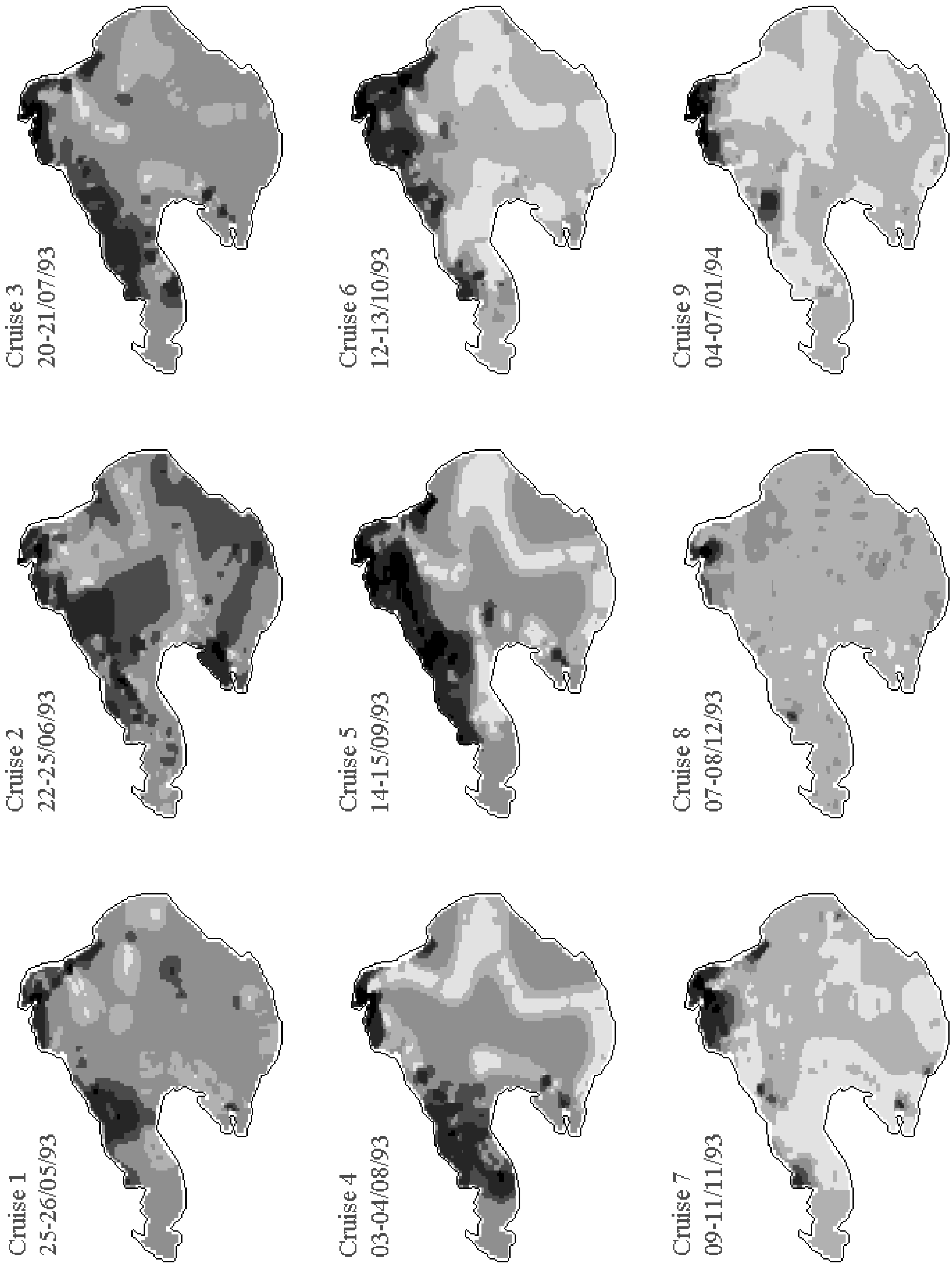


Figure 24.(cont.) Interpolated S.E. of average Nitrate distributions by cruise

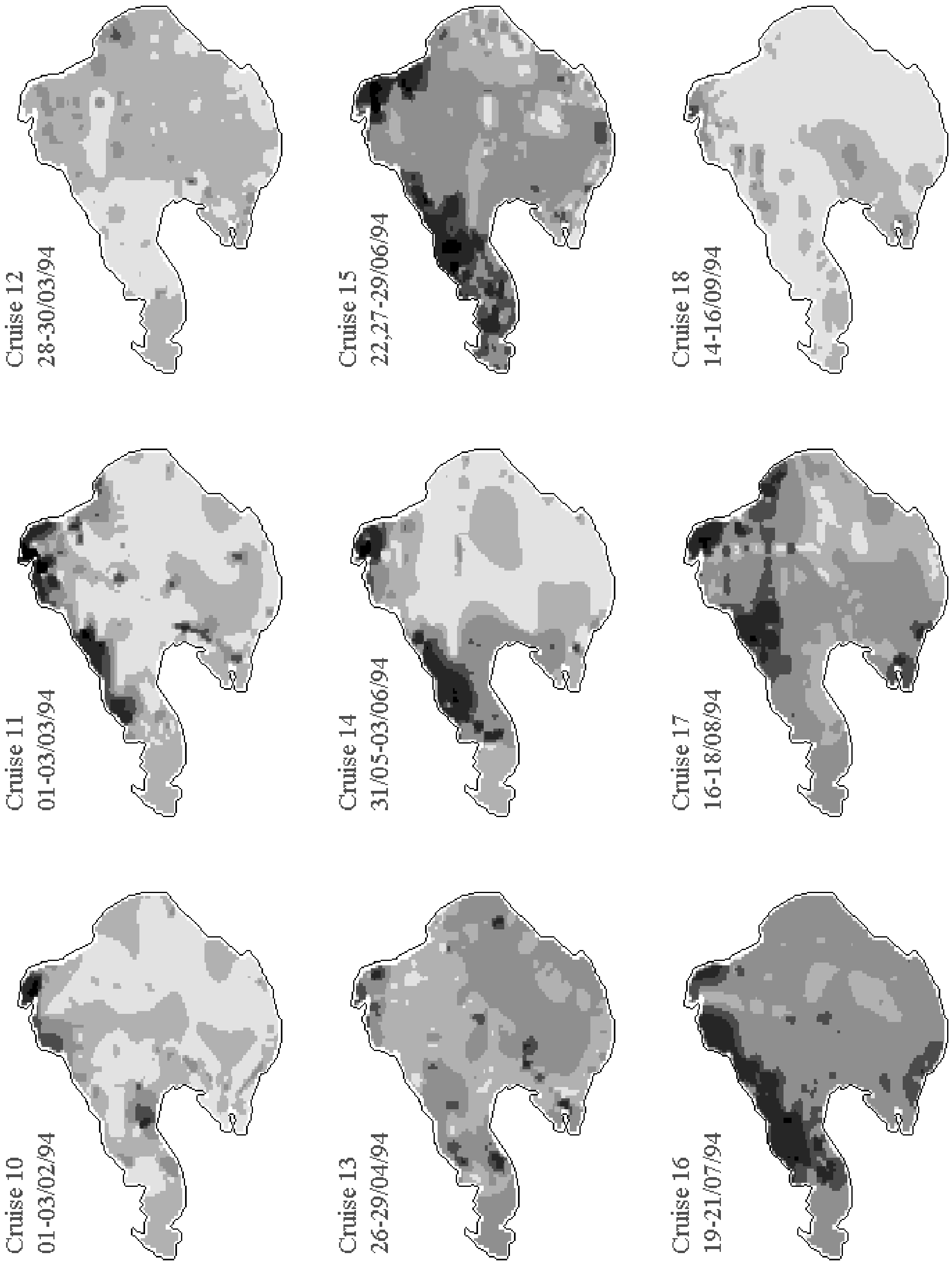


Figure 24.(cont.) Interpolated S.E. of average Nitrate distributions by cruise

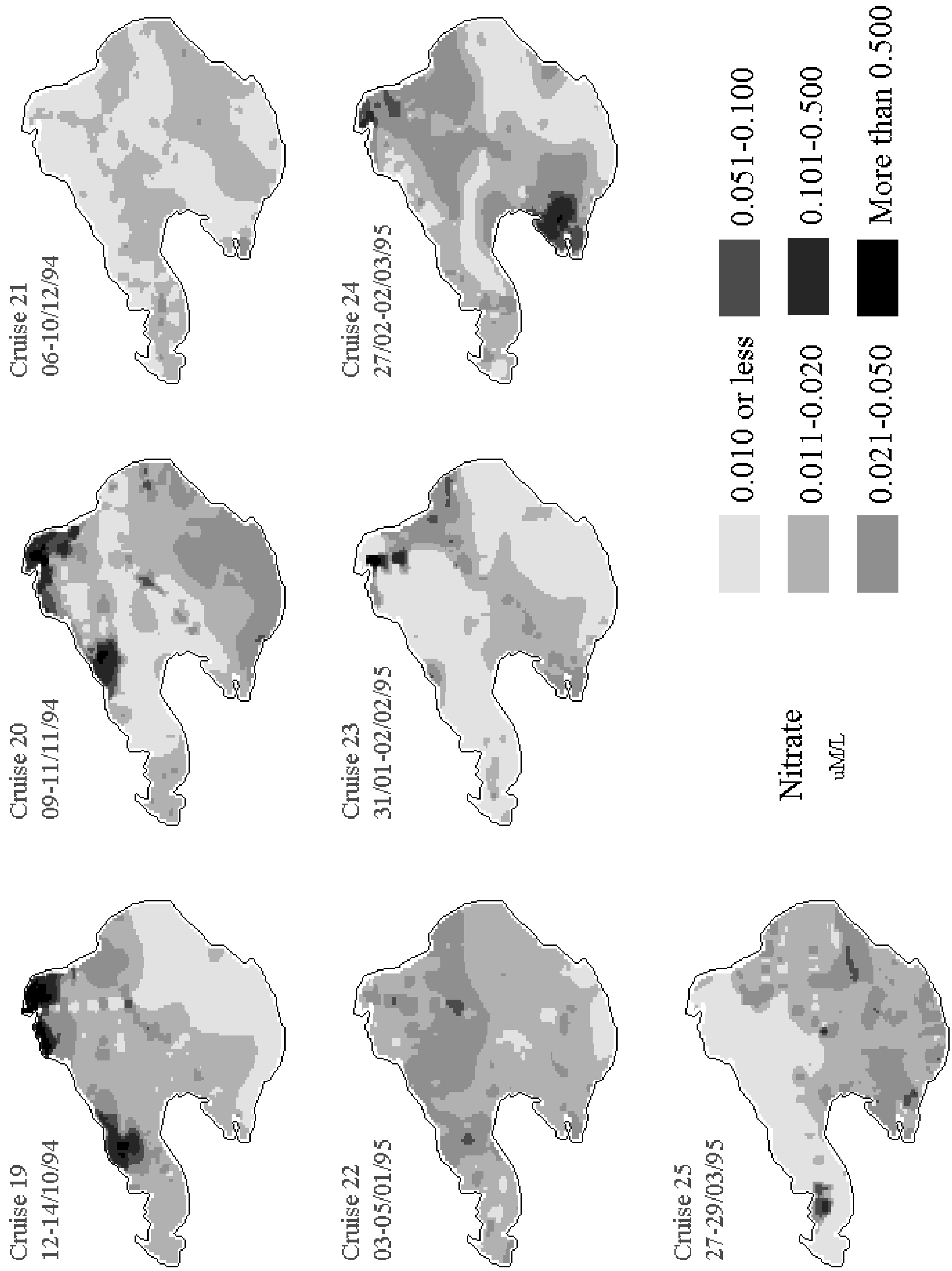


Figure 25. Interpolated S.E. of average Phosphate distributions by cruise

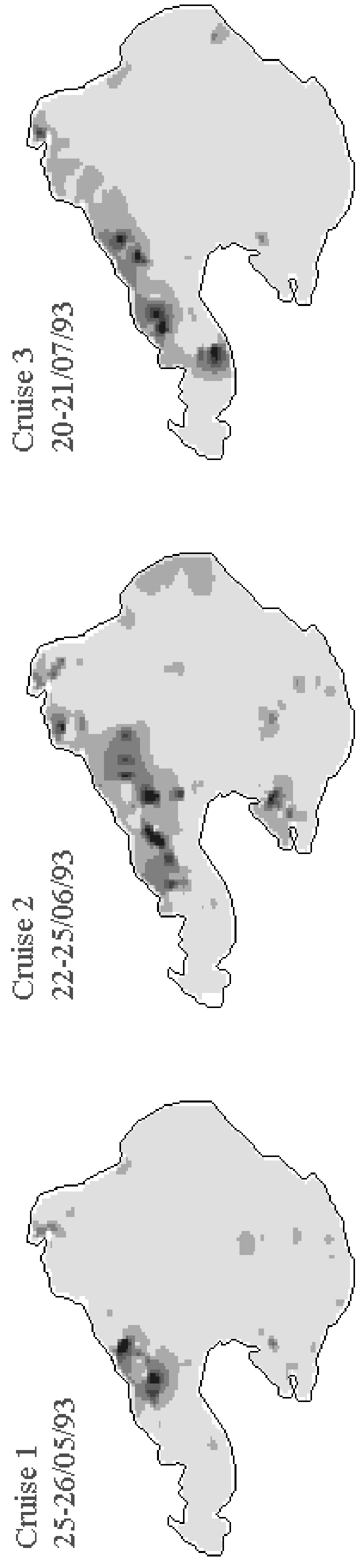


Figure 25.(cont.) Interpolated S.E. of average Phosphate distributions by cruise

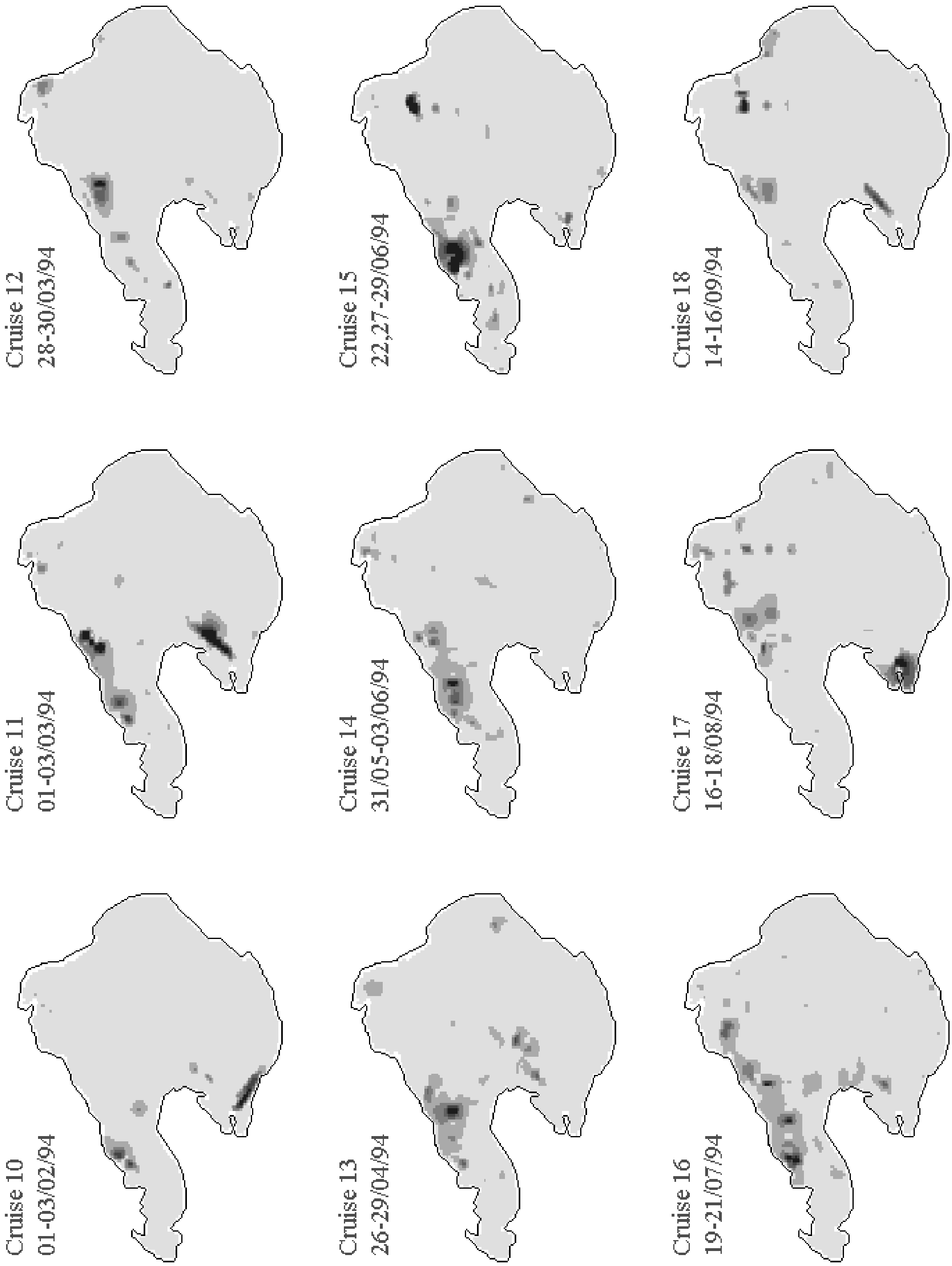


Figure 25.(cont.) Interpolated S.E. of average Phosphate distributions by cruise

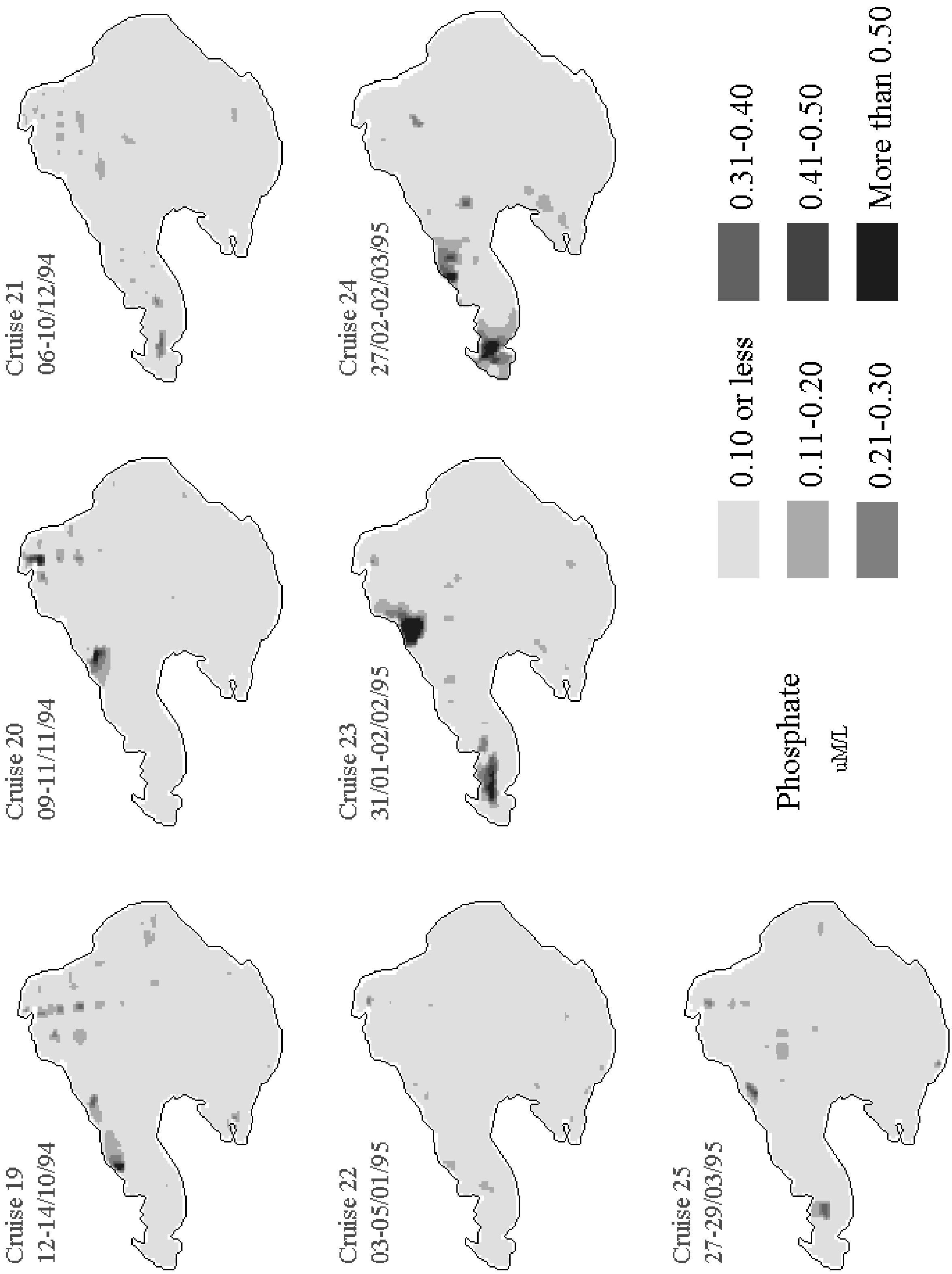


Figure 26. Interpolated S.E. of average Silicate distributions by cruise

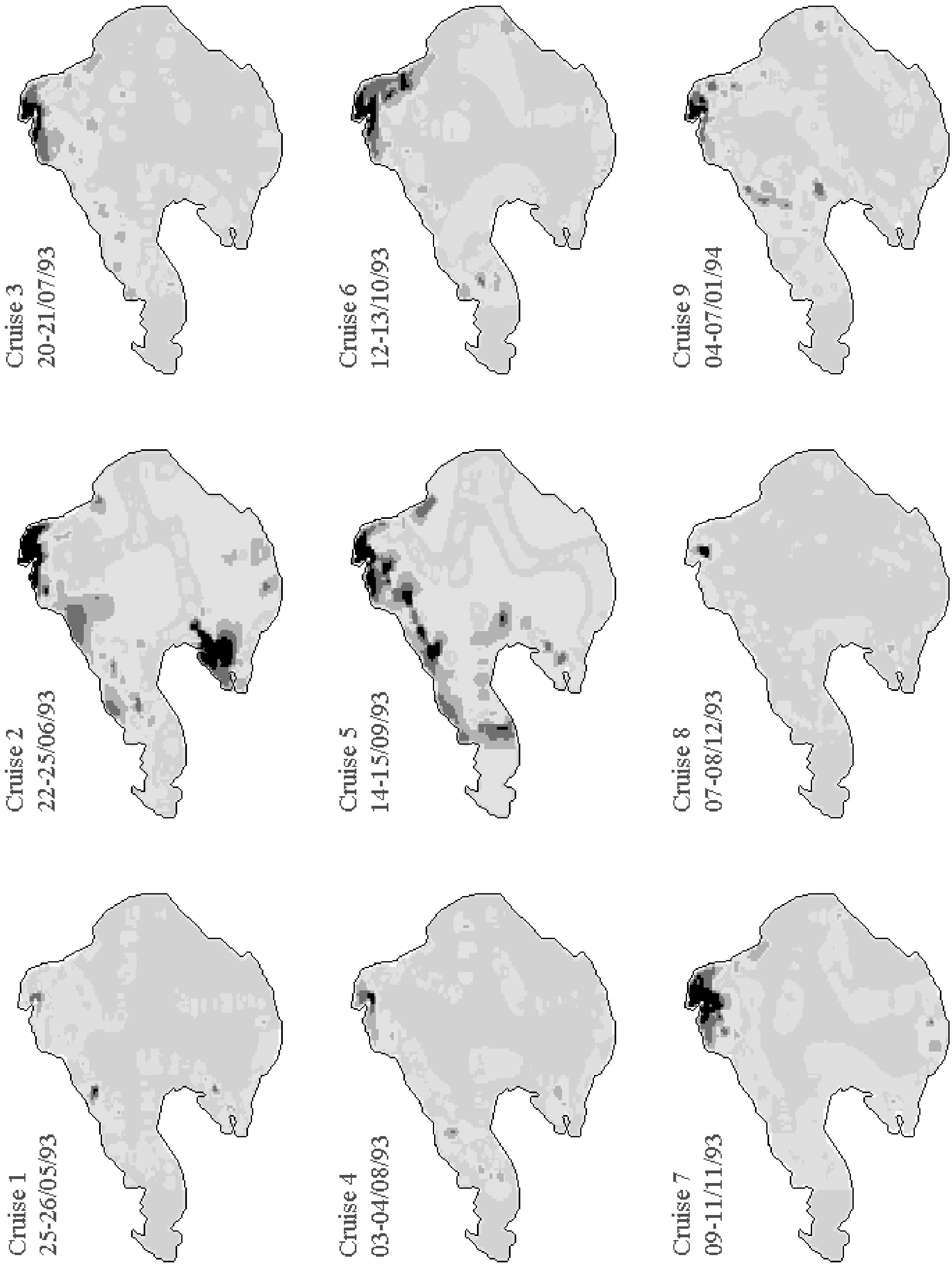


Figure 26.(cont.) Interpolated S.E. of average Silicate distributions by cruise

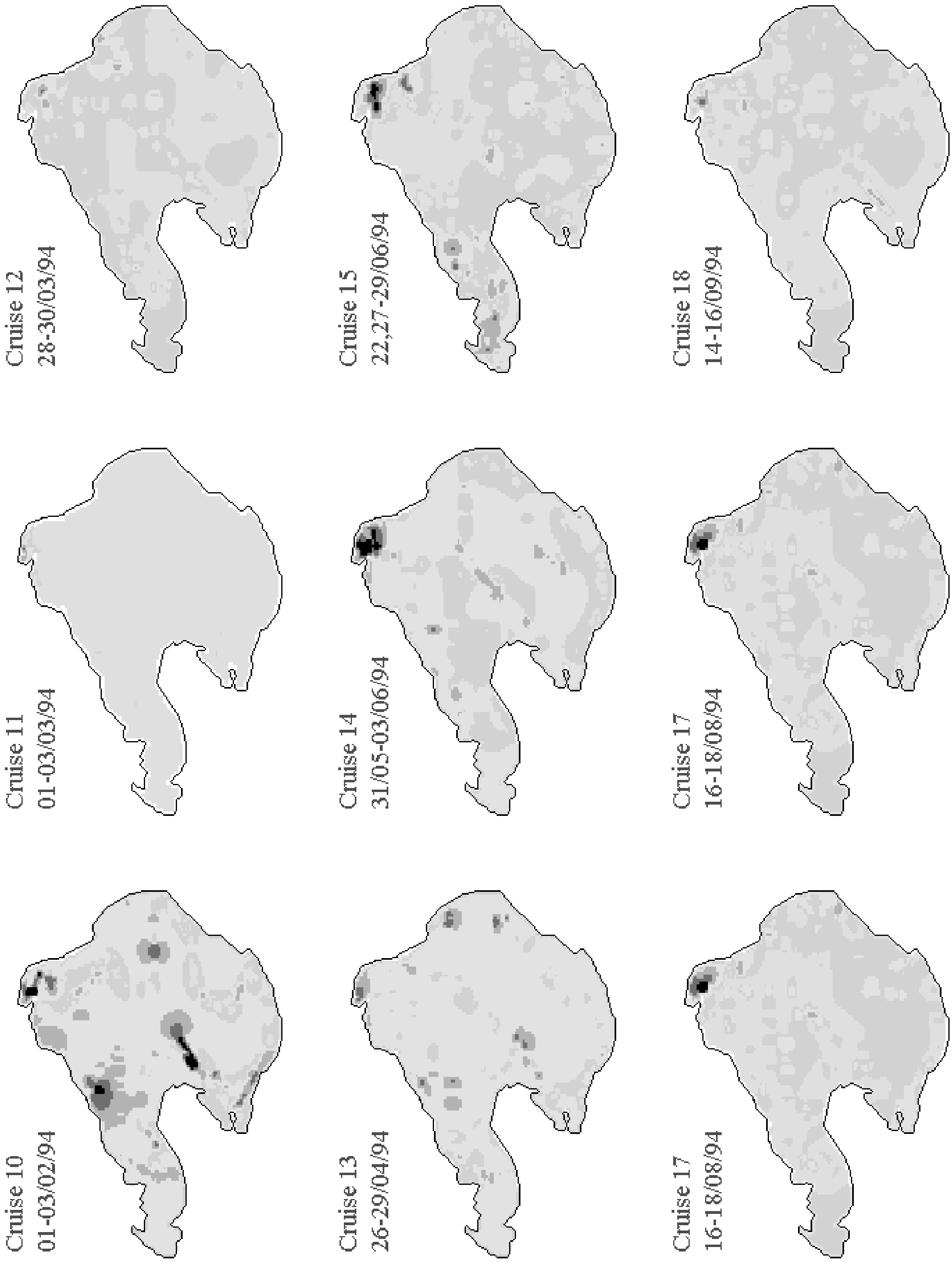


Figure 26.(cont.) Interpolated S.E. of average Silicate distributions by cruise